Stock-Oil Comovements Through Fear, Uncertainty, and Expectations: Evidence from Conditional Coskewness and Cokurtosis

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Abstract

This paper investigates the dependencies and comovements between S&P 500 and WTI futures price by means of time-varying conditional beta, coskewness, and cokurtosis from April 1983 to December 2021. The conditional beta, coskewness, and cokurtosis show a new phase of stronger relationship between the two markets since the 2008 global financial crisis (GFC). WTI shows greater sensitivity to S&P 500's volatility and tail events than its counterpart. Relying on time-varying parameter vector autoregression (TVP-VAR), and employing three daily macroeconomic indicators including VIX (representing fear), economic policy uncertainty (representing uncertainty), and expected business condition index (representing expectation), the impulse response of the stock-oil conditional comoments to macroeconomic conditions are analyzed thoroughly. In particular, since GFC: (i) S&P 500 beta shows much more stronger positive response to VIX shocks, while on the contrary WTI shows larger and stronger negative response to VIX, (ii) conditional cokurtosis impulse responses show patterns similar to betas, with economic uncertainty having greater impact on the stock-oil dependencies, (iii) overall shocks from VIX, economic uncertainty, and expected business condition, take longer to impact WTI conditional comoments, while they tend to be more transient for S&P 500, and (iv) shocks to expectations on business condition creates stronger, more volatile and, more asymmetric response across all conditional comoments measures than VIX or economic policy uncertainty.

Keywords: Stock-Oil dynamics, Volatility modeling, Conditional Coskewness, Conditional Cokurtosis, Energy market, Macroeconomic effect

1 Introduction

There is a rich literature on the oil price dynamics with economic variables and financial markets, due to the importance and fundamental effect of oil price in economy. There are many papers (Singleton, 2014; Christoffersen & Pan, 2018; Christoffersen et al., 2019 among others), showing that from 2004 onward - the period termed as "financialization" of commodity markets - the links between stock market and commodities have increased vastly. Christoffersen et al. (2019) show that commodity volatility correlations have set a new mean since that time and they are strongly related to stock market volatility (more than returns). They state that "the principal components in commodity futures volatility appear to be strongly related to volatility in other asset markets including the U.S. equity market". Christoffersen & Pan (2018) even find that oil shocks create funding constraints in financial intermediaries.

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A large portion of these studies are conducted by a model-free approach directly by VAR. On the other side, models based on higher moments have not received enough attention. Fang & Lai (1997) in a seminal work state that, systemic skewness and kurtosis are a contributing factor in explaining expected return, in addition to the systemic variance. Smith (2007) finds that coskewness is significant factor in explaining the cross-section of asset returns in a three-moment CAPM model. He points to the time-varying nature of coskewness risk premia, and finds that a pricing model equipped with coskewness factor outperforms conventional three-factor or four-factor models.

Several other works including, Vanden (2006), Guidolin & Timmermann (2008), J. Yang et al. (2010), C.-Y. Yang & Chen (2009), Chang et al. (2013), and Chan et al. (2018) strongly support the role of higher co-moments in financial assets' return and risk premia. Chiang (2016) studies bond skewness and coskewness, emphasizes the importance of modeling coskewness. He also finds that skewness, and coskewness are time-varying. Högholm et al. (2011) show that factor loadings on coskewness and cokurtosis are time-varying, asymmetric, and state dependent. In this regard, they oppose employing linear, or regime switching models, and mention that separating the market conditions to bull and bear cannot capture the asymmetric effect of higher moments effects. These findings are in line with the fact that, during the regimes the conditional correlations and variances are supposed to remain constant.

Xiong et al. (2014) show that tail risk is compensated for equity funds unlike volatility (beta). Heath (2019) states that oil price shocks have strong effect on economic activities. He points to the fact that previous models developed in VARs, miss variations in oil price risk. Oil price premium is largely time-varying and pro-cyclical. The oil shocks create large impact during economic expansion. He also pinpoints an important of using futures price which is, capturing all types of demands, from speculators to true hedgers, so oil futures price is truly representative of all oil market agents. It is a stylized fact that futures market (especially for oil) is deeper (more active than spot market) and has a time-varying term structure which helps forecast spot prices. Christoffersen et al. (2021) propose a strategy to estimate the price of co-skewness and co-kurtosis, emphasizing the inefficiently of conventional approaches that rely on two-stage cross-sectional or Fama-MacBeth regressions. Their model is based on index options. In two works by Bollerslev et al. (2009) and Bollerslev et al. (2014), variance risk premium is identified as the best predictor of market returns. Dittmar (2002) states that kurtosis measure the likelihood of extreme values and co-kurtosis captures the sensitivity of asset returns to extreme market return realizations. In this regard, kurtosis includes both systemic and non-systemic risk (noisy part), while cokurtosis is truly representative of the systemic risk.

To the knowledge of the author, there is no research studying the stock-oil comovements and dependencies by means of higher moment conditional coskewness, and cokurtosis. In this regard, this paper tries to fill the gap for the fundamental stock-oil literature and their tail dependencies using conditional coskewness and cokurtosis, with specific focus on the interactions between these assets during a long period from April 1983 up to Dec 2021. Unlike most studies which employ spot prices for WTI, we utilizes futures prices, which are truly sensitive to shocks and news, a good characteristics of futures market, and a perfect setting in terms of risk hedging. Moreover we study the effect of macroeconomic shocks on the conditional comoments using three famous indices, CBOE VIX representing fear level, Baker et al. (2016) economic policy uncertainty (EPU) which represents uncertainty about market condition, and the thorough Aruoba et al. (2009) index of expectation on real economic condition (ADS henceforth). We emphasize the unique feature of the three macroeconomic indicators in having high

frequency daily data. A feature which is very important in studies on volatility modeling, volatility/shock spillover, and systemic risk.

The fundamental literature on stock-oil relationship finds that since the 2008 global financial crisis (GFC) the dynamics of the stock market and crude oil price has changed from a negative relationship to a positive relationship. Several papers attribute the phenomenon to aggregate growth shocks (Hitzemann, 2016), slow down in global economic activity (Bernanke, 2016), the sovereign wealth funds speculative activity (Mohaddes & Pesaran, 2017), or oil demand shocks (Ready, 2018). The findings of this paper, based on conditional beta, coskewness, and cokurtosis provides further evidence for the this phenomena. It is essential to note that, unlike past studies, this paper employs economically sound measures to talk about the comovements, sensitivity, and spillovers. Our measures of comovements are based on realized volatility calculated on historical data, have higher frequency (daily data), and are time-varying. The coskewness and cokurtosis can be viewed as measure of sensitivity to tail events as well. We calculate the time-varying conditional betas, coskewnesses, and cokurtoses based on asymmetric dynamic conditional correlation (ADCC) GJR-GARCH (Glosten et al., 1993, Engle, 2002,Cappiello et al., 2006) accommodating the asymmetric responses between and within the financial series. Then to study the impact of different phases of business cycles, macroeconomic variables, and financial crashes on the conditional comoments, we rely on time-varying parameter vector autoregression (TVP-VAR) by Primiceri (2005) to analyze the time-varying impulse responses of the three macroeconomic indicators.

The contribution of this paper are: We calculate alternative, economically sound measures of comovements for stock-oil relationship. The conditional beta, coskewness and cokurtosis have a time-varying nature (for both equity and oil market) and show a new phase of stronger relationship between the two markets since the 2008 global financial crisis (GFC). WTI shows greater sensitivity to S&P 500's volatility and tail events than its counterpart based on the three conditional comoments. Since GFC: (i) S&P 500 beta shows much more stronger positive response to VIX shocks, while on the contrary WTI shows larger and stronger negative response to VIX, (ii) conditional cokurtosis impulse responses show patterns similar to betas, with economic uncertainty having greater impact on the stock-oil dependencies, (iii) shocks from VIX, economic uncertainty, and expected business condition, take longer to impact WTI conditional cokurtosis, while they tend to be more transient for S&P 500 cokurtosis, and (iv) shocks to expectations on business condition creates stronger (ADS), more volatile and, more asymmetric response across all conditional comoments measures than VIX or economic policy uncertainty. The paper is organized as follows. Section 2 describes the data, discusses the theoretical model as well as the empirical approach. Section 3 presents the results and discusses. Section 4 concludes.

2 Method & Data

In our empirical analysis, we first model the conditional coskewness and cokurtosis between WTI continuous time futures price index[1], and S&P 500 index from April 1983 to December 2021 using daily data. Table 1 present the descriptive statistics for our datasets. We calculate the daily returns as $Ln(\frac{P_t}{P_{t-1}})$ where P_t refers to index price at time t. Note that, all returns are in excess of the one-month TBill rate.

[1]Based on nearest-to-maturity price

[2]ADF test with constant, linear and quadratic trend

Table 1: Descriptive statistics of the dataset

	Mean	SD	Skewness	Robust Skewness	Kurtosis	Robust Kurtosis	ES	Jarque-Bera	ADF[2]
S&P500	0.00035	0.011	-1.22	-0.035	30	4.41	0.027	FALSE	-17.5
WTI Futures	0.00010	0.024	-1.90	-0.028	50.6	3.99	0.056	FALSE	-17.8

We employ three important macroeconomic indices including CBOE volatility index (VIX) (also know as gauging stock market fear level), Baker et al. (2016) EPU, and Aruoba et al. (2009) real business conditions index[3]. The EPU uncertainty index[4] developed by Baker et al. (2016) which is a true uncertainty measure, this news-based index uses newspaper archives from the Access World News Bank service on words related and/or associate with the word uncertainty. Several studies show significant negative relationship between EPU and macro variables like growth rates and employment rates. EPU is shown to be able to explain large fluctuation for equity market like S&P 500. The Aruoba et al. (2009) ADS daily index[5] is based on expectation on real business conditions which is computed on economic variables like: weekly initial jobless claims; monthly payroll employment, monthly industrial production, monthly real personal income less transfer payments, monthly real manufacturing and trade sales, and quarterly real GDP.
The index has a zero mean, with positive values as progressively better-than-average conditions, and negative values as progressively worse-than-average conditions. Figure 1 shows the macroeconomic indices series. Note that in studying the impact of macroeconomic factors' on the conditional coskewness and cokurtosis, the data starts from 1990 due to the availability of the macroeconomic data. The giant impact of COVID-19 in the beginning of the 2020 is distinct and phenomenon in figure 1 given. Note that, the unique feature of the three macroeconomic indicators is that they are available at daily frequencies, which complements our stock-oil daily dynamics.

2.1 Conditional Coskewness & Cokurtosis: Alternative Tail Risk Exposure

The importance of higher co-moments in asset pricing primarily was introduced by Samuelson (1975), and Rubinstein (1973), which later on becomes a stylized fact in asset pricing models proposed by Kraus & Litzenberger (1976), Fang & Lai (1997), and Harvey & Siddique (2000). CAPM a two-moment (mean-variance) asset pricing model states that, given the risk free rate r_f, the expected gross return $R_{i,t}$ of an asset i at time is generated by the expected market premium (or excess return) $R_{m,t}$ through asset i's conditional covariance with the market scaled by the market's variance $\sigma^2_{m,t}$ that is :

$$E(R_i) - r_f = \frac{Cov(R_m, R_i)}{Var(R_m)} E_t(R_m) \tag{1}$$

Note that the covariance $Cov(R_m, R_i)$ is equal to $E[(R_i - \bar{R}_i)(R_m - \bar{R}_m)]$ with $\bar{R}_i$ and $\bar{R}_m$ referring to the mean value of the asset i's, and market returns. Quite often $\frac{Cov(R_m, R_i)}{Var(R_m)}$ is defined as the conditional $\beta_{i,t}$ of the asset i with the market. Now let's provide a broader view of the CAPM foundations. In the absence of arbitrage given the information set Ω_t available to an investor, and at time t the asset i's gross return $r_{i,t+1}$, the stochastic discount factor m_{t+1} is

[3]Note that the raw historical series for all indices are stationary at 1%
[4]The data is available at federal reserve bank of Saint Louis
[5]The data is available at federal reserve bank of Philadelphia

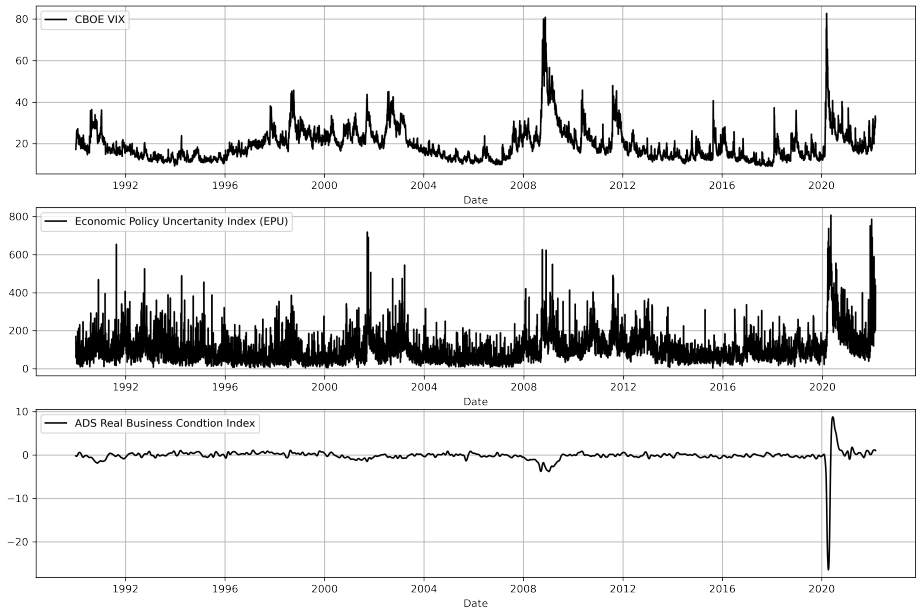


Figure 1: Macroeconomic indices

defined as (Cochrane, 2009):

$$E[m_{t+1}R_{i,t+1} \mid \Omega_t] = 1 \tag{2}$$

If the canonical assumption of CAPM are met including Gaussian distribution of returns, or quadratic utility function for an investor (among other assumptions), then the stochastic discount factor (SDF) m_{t+1} can be formulated as a linear function of market return, or:

$$m_{t+1} = a + b_t R_{m,t+1} \tag{3}$$

However given the evident failure of assumption of CAPM model including the Gaussian distribution of the returns, quadratic utility function[6] etc, the SDF has a nonlinear relationship with the market return $R_{m,t+1}$, and can be specified with a third-order polynomial of market return as:

$$m_{t+1} = a + b_t R_{m,t+1} + c_t R^2_{m,t+1} + d_t R^3_{m,t+1} \tag{4}$$

Following Fang & Lai (1997) the CAPM is extended to account for the coskewness and cokurtosis in a four-moment pricing model:

$$E(R_i) - r_f = \zeta_1 Cov(R_m, R_i) + \zeta_2 Cov(R^2_m, R_i) + \zeta_3 Cov(R^3_m, R_i) \tag{5}$$

Where $Cov(R^2_m, R_i)$, and $Cov(R^3_m, R_i)$ capture the time-varying conditional coskewness and conditional cokurtosis respectively. It is noteworthy to state that conditional coskewness and cokurtosis capture what conventional measures of return comovements cannot, because they

[6]Quadratic utility requires investors to have increasing absolute risk aversion, which is counter intuitive

are based on the comovements and sensitivity to the tail events similar to the expected shortfall. It is evident that in this equation, our measures of coskewness and cokurtosis are not scaled, so in order to arrive at a correct inference of for the relevant tail risk measures, we must scale these covariances. In this regard we have that:

$$Beta_i(\beta_i) = \frac{Cov(R_m, R_i)}{Var(R_m)} \quad Coskewness_i(\Gamma_i) = \frac{Cov(R_m^2, R_i)}{[Var(R_m)]^{1.5}} \quad Cokurtosis_i(\Lambda_i) = \frac{Cov(R_m^3, R_i)}{[Var(R_m)]^2} \tag{6}$$

From left, we get the famous asset's β after dividing the $Cov(R_m, R_i)$ by the market variance $Var(R_m)$, and for the conditional coskewness and cokurtosis of the asset i with the market, we divide $Cov(R_m^2, R_i)$ and $Cov(R_m^3, R_i)$ by the market's conditional skewness $[Var(R_m)]^{1.5}$ and kurtosis $Var(R_m)^2$.

Now after scaling covariances we arrive at the following model which is starting point in our empirical investigation:

$$E(R_i) - r_f = \zeta_1' \beta_{i,t} + \zeta_2' \Gamma_{i,t} + \zeta_3' \Lambda_{i,t} \tag{7}$$

GARCH approach, is superior to regime switching model employed in many of the past studies, like Guidolin & Timmermann (2008), J. Yang et al. (2010), Chiang (2016), and Chan et al. (2018) in that the correlations covariance are time-varying and not constant. Francq & Zakoian (2019) state that while Markov switching regime models are flexible, they are only an approximation of the data generating process. Additionally in our empirical model, we consider the role of asymmetries (leverage effect) within and in-between the assets, in addition to our Student's t distribution in estimations given the fat-tailed return series[7]. None of the above studies address those issues in their work. Our method in this paper is also superior to rolling regression, in that we take into account the stochastic nature of the conditional comovements in the time-varying covariances, and there is no arbitrary choice of an interval, nor neglecting some part of the data as in rolling regression methods. As follows, we build the varibales based on realized volatility (rather option prices) which is more informative of the underlying data generating process.

In order to compute the time-varying higher comoments, we employ asymmetric dynamic conditional correlation ADCC-MGARCH process by Cappiello et al. (2006) and a GJR(1,1) of Glosten et al. (1993) for individual volatility processes for each time series in the ADCC process. In this regard we start by computing the following mean equation model for each time series:

$$R_{wti,t} = \phi_{wti} + \epsilon_{wti,t} \tag{8}$$

$$R_{sp,t} = \phi_{sp} + \epsilon_{sp,t} \tag{9}$$

Where ϕ_{wti} and ϕ_{sp} are the relevant constants and $\epsilon_{wti,t}$ and $\epsilon_{sp,t}$ are the relevant innovations. We apply $ADCC - GJR - t$ on this equation, in order to compute the $Cov(R_{sp}, R_{wti})$. To compute the $Cov(R_{sp}^2, R_{wti})$ and $Cov(R_{sp}^3, R_{wti})$, we substitute in the above equation R_{sp}^2 and R_{sp}^3 respectively in equation 8. ADCC model is parsimonious in addition to being able to capture the effects of asymmetry in comovements between the time series. Also, a multivariate Student's t distribution accounts for the fat-tailed series. We scale those estimated conditional covariances by the conditional variance of the S&P 500 using a univariate GJR(1,1).

[7]Francq & Zakoian (2019)encourage the use of heavy-tailed marginal distributions when estimating long return series with GARCH processes, to avoid a possible spurious strong volatility persistence

In order to compute the time-varying dynamics of the covariances in this study, we employ ADCC-GJR-t Cappiello et al. (2006). The ADCC model is built on the DCC model of Engle (2002). Dynamic conditional correlation models, are an intelligent response to the heteroskedasticity of the disturbances. The asymmetric DCC can capture the potential asymmetries between comovements of the assets, in addition to have stronger statistical properties. The asymmetric GJR-GARCH process of Glosten et al. (1993) reads as:

$$h_{i,t}^2 = \omega_i + \alpha_i e_{i,t-1}^2 + \beta_i' h_{i,t-1}^2 + \gamma_i e_{i,t-1}^2 I_{i,t-1} \tag{10}$$

$$where \qquad I = \begin{cases} 1 & if\ e_{i,t-1} < 0 \\ 0 & Otherwise \end{cases} \tag{11}$$

Now that we have computed the conditional variances in the previous step, we need to compute the standardized errors by $\varepsilon_{i,t} = \dfrac{\epsilon_{i,t}}{h_{i,t}}$ and the time-varying covariance matrix H_t. We state that the vector of time-varying errors follows $\mathbf{E}_t \mid \Delta_{t-1} \sim Student's\ t(0, H_t, v)$ where the time-varying covariance matrix $H_t = D_t P_t D_t$ consists of the time-varying correlation matrix P_t and the time-varying diagonal matrix of the standard deviations D_t. The time-varying correlation matrix P_t reads as:

$$P_t = diag{Q_t}^{-1} Q_t diag{Q_t}^{-1} \tag{12}$$

where:

$$Q_t = (\bar{P} - a^2\bar{P} - b^2\bar{P} - g\bar{N}) + a^2(\varepsilon_{t-1}\varepsilon_{t-1}') + g^2(n_{t-1}n_{t-1}') + b^2 Q_{t-1} \tag{13}$$

Where $\bar{P} = E[\varepsilon_{t-1}\varepsilon_{t-1}']$ is the unconditional correlation matrix of the standardized errors, and $\bar{N} = E[n_{t-1}n_{t-1}']$. Further the effect of negative shock is absorbed by the variable n_t which can be expressed as a Hadamard product with the following characteristics:

$$n_t = I[\varepsilon_{t-1}] \circ \varepsilon_{t-1} = \begin{cases} \varepsilon_t & \varepsilon_t < 0 \\ 0 & \varepsilon_t \geq 0 \end{cases} \tag{14}$$

Where $I[\varepsilon_{t-1}]$ is a $K \times 1$ indicator function. The positive definiteness of the Q_t is guaranteed only if $(a^2 + b^2 + \frac{1}{2}g^2) \leq 1$. We employ a multivariate Student's t for to estimate the parameters of the time-varying correlation matrix Q_t. In the following table we report the estimated parameters of a GJR(1,1) process for each of the parameters.

Table 2: Volatility equation parameters

	S&P500	*p-value*	WTI futures	*p-value*	$S\&P500^2$	*p-value*	$S\&P500^3$	*p-value*	WTI^2 futures	*p-value*	WTI^3 futures	*p-value*
$\phi * 10^4$	0.02	0.00	0.00	0.57	0.1	0.89	0.1	0.13	1.9	0.14	0.4	0.51
$\omega * 10^4$	0.02	0.00	0.02	0.31	0.02	0.00	0.3	0.01	0.6	0.00	144	0.00
$ARCH\ (\alpha)$	0.015	0.03	0.062	0.00	0.057	0.00	0.027	0.20	0.07	0.00	0.068	0.00
$GARCH\ (\beta')$	0.896	0.00	0.911	0.00	0.92	0.00	0.891	0.00	0.924	0.00	0.826	0.00
$GJR\ (\gamma)$	0.131	0.00	0.047	0.00	-0.004	0.79	0.148	0.00	-0.039	0.49	0.167	0.00

2.2 Conditional Coskewness and Cokurtosis Risk Premia: GMM approach

After estimating the empirical model, we employ GMM to study the effect of the higher conditional co-moments for our two variables. It is crucial to mention that, as our variables are generated ones, there are a few methods to deal with the endogenous inherent in our conditional

higher co-moments. In this regard, some authors also orthogonalize the variables onto one another, which is not perfect practice compared to the GMM. Given that we are interested in studying the response of conditional comovements to macroeconomic variables through time, we do not impose any further constraints or limits on our initial dual multivariate GARCH processes in the equation 8 and 9. In the later section, we employ local time-varying parameter vector autoregression (TVP-VAR) which is a perfect tool for studies on potentially nonlinear comovements and time-varying impulse responces.

2.3 Fear, Uncertainty, and Expectations: TVP-VAR with stochastic volatility

In order to study the impact of the three daily macroeconomic variables on conditional coskewness and a TVP-VAR (Primiceri, 2005) is employed. The method belongs to the non-linear VAR class (Primiceri, 2005, Kilian & Lütkepohl (2017)) and is able to capture the asymmetric effect of positive vs negative structural shocks either by the state of the economy or by the variables and lags. Additionally this model is developed to provide time-varying VAR coefficients (based on some stochastic process) and impulse response functions (IRFs) which is suitable to our analysis of the conditional higher comoments that encompasses several decades of different economic policies and boom/bust. The evident existence of the different economic regimes, change in the condtional volatility of both oil and stock market (refer to the results) all refer to the smooth structural change (Primiceri, 2005, Nakajima et al., 2011 ,Del Negro & Primiceri, 2015; Kilian & Lütkepohl, 2017). Following Primiceri (2005) the TVP-VAR model is derived from structural VAR model, and reads as follow for a multivariate case:

$$\boldsymbol{Y}_t = \nu_t + A_{1,t}\boldsymbol{Y}_{t-1} + ... + A_{p,t}\boldsymbol{Y}_{t-p} + u_t \tag{15}$$

Where $\boldsymbol{Y}$ is a $n \times 1$ vector of endogenous variables, ν_t is a $n \times 1$ vector of coefficients that multiply the constant terms, the coefficients $A_{1,t}, i = 1, ..., p$ are dependent on time. The error term u_t is assumed to be a zero-mean white noise process with time-varying covariance matrix, that is $u_t \sim (0, \Sigma_{u,t})$. To facilitate structural analysis, the error covariance is decomposed to:

$$\Sigma_{u,t} = B_t^{-1}\Sigma_{w,t}B_t^{'-1} \tag{16}$$

Where $\Sigma_{w,t} = diag[\sigma_{1,t}^2, ..., \sigma_{k,t}^2]$ is a diagonal matrix with the variances of the structural errors, and B_t^{-1} may be a lower-triangular matrix as follow:

$$B_t = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ b_{21,t} & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ b_{k1,t} & \cdots & b_{kk-1,t} & 1 \end{bmatrix}$$

Restrictions on B_t can be used to uniquely identify the structural shocks or $w_t = B_t u_t$, and using it, we can rewrite the model in structural form as:

$$\boldsymbol{Y}_t = \nu_t + A_{1,t}\boldsymbol{Y}_{t-1} + ... + A_{p,t}\boldsymbol{Y}_{t-p} + B_t^{-1}u_t \tag{17}$$

If we gather all all the VAR slope coefficients in the vector $\boldsymbol{\alpha}_t = vec[\nu_t, A_{1,t}, ..., A_{p,t}]$ and the unrestricted elements of the B_t in $\boldsymbol{b}_t = [b_{21,t}, b_{31,t}, b_{31,t}, \cdots, b_{k1,t}, \cdots, b_{kk-1,t}]'$ then the vector $\boldsymbol{b}_t$ is the $\frac{1}{2}K(K-1)$-dimensional vector of elements below the main diagonal of B_t which is row-wised such that the parameters for each individual equation are grouped together. Now having $\sigma_t = [\sigma_{1,t}, \cdots, \sigma_{k,t}]'$ as the vector of w_t's standard deviations, we can specify the dynamics of the

time-varying vectors of coefficients as random walk processes for $\boldsymbol{\alpha}_t$ and $\boldsymbol{b}_t$, and a geometric random walk for σ_t, restating, the model allows for stochastic volatility with considerable persistence:

$$\boldsymbol{\alpha}_t = \boldsymbol{\alpha}_{t-1} + \eta_t^{\alpha} \tag{18}$$

$$\boldsymbol{b}_t = \boldsymbol{b}_{t-1} + \eta_t^{b} \tag{19}$$

$$log\sigma_t = log\sigma_{t-1} + \eta_t^{\sigma} \tag{20}$$

Summing up, the co/variance matrix error terms of the model equations is block diagonal as:

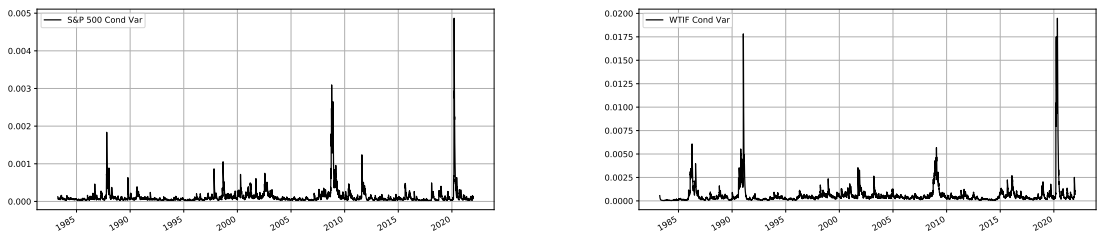
$$Cov \begin{bmatrix} w_t \\ \eta_t^{\alpha} \\ \eta_t^{b} \\ \eta_t^{\sigma} \end{bmatrix} = \begin{bmatrix} \Sigma_{w,t} & 0 & 0 & 0 \\ 0 & \Sigma_{\alpha} & 0 & 0 \\ 0 & 0 & \Sigma_b & 0 \\ 0 & 0 & 0 & \Sigma_{\sigma} \end{bmatrix}$$

Where Σ_{α}, Σ_b, and Σ_{σ} are the positive definite covariance matrices of η_t^{α}, η_t^{b}, and η_t^{σ} respectively. Note that, the error terms are independent of one another. Assuming $Z_{t-1} \equiv (1, \boldsymbol{Y}_t, \cdots, \boldsymbol{Y}_{t-p})'$ the initial VAR in equation 15 becomes:

$$\boldsymbol{Y}_t = (Z'_{t-1} \otimes I_k)\boldsymbol{\alpha}_t + u_t \tag{21}$$

With the symbol $\otimes$ referring to Kronecker product. In the above equation (eq. 21) and the random walk process of α_t (eq. 18) are basically a state-space model with measurement equation (eq. 21) and transition equation (eq. 18). Generally there are two ways to estimate this model, one is maximum likelihood estimation, and Bayesian estimation based on the Markov Chain Monte Carlo (MCMC), which the latter is employed given the issues with maximum likelihood estimation.

3 Result & Discussions



(a) S&P 500

(b) WTI futures

Figure 2: Conditional variances 1983-2021

Figure 2 shows the conditional variances for S&P 500 and WTI futures. The giant impact of the recent financial crisis relevant to COVID-19 pandemic period is significantly greater than any crisis during the last four decades for both of the variables, especially for S&P500. While

COVID-19 effect shock on oil volatility is comparable to the Iraq-Kuwait war in early 1990s.

Figure 3, and 4 exhibits the conditional beta, coskewness and cokurtosis of S&P 500 and WTI futures respectively. Figure 4 shows that from 1983 to global financial crisis (GFC) of 2008, WTI's conditional beta has had a zero mean overall with two salient negative fluctuation around the 1987 black Monday crisis and Iraq-Kuwait war, while after GFC the sensitivity of the WTI to the S&P 500 finds a new positive mean, such that whenever the political climate becomes unstable around the world this sensitivity increases positively, representing the fact that the two market are getting more connected than before. In addition, this emerging pattern is a source of concern for investors who used to benefit from diversification by the two asset classes.. We do not witness any effect of financialization era in our study. The conditional cokurtosis of WTI (figure 4) evidences the same pattern. The reaction of WTI to S&P 500 tail events is more stronger since GFC, and the strongest as of COVID-19 emergence in early 2020.
Figures 5-7 show the impulse responses of our conditional comoments for S&P 500 (panel a), and WTI futures (panel b) to a 1% permanent increase in the positive shock in VIX, economic policy uncertainty (EPU), and the expectation on business condition (ADS) indicators. There are three intervals, a 6-day response, a 12-day response and a 24-day response [8]. Overall, the impulse responses show time-varying and sometimes asymmetric behavior across our long dataset. Looking at figures 5-7, we realize that there can three distinct periods of economic activity, one from 1990-2000, another from 2000-2008, the other initiates from 2008. Another pattern relates to the responses of WTI conditional comoments to shocks from VIX, EPU, and ADS. Compared to S&P 500, the 24-day response is more visible and stronger than 6-day responses, while S&P 500 conditional comoments respond much quicker to shocks from the macroeconomic variables. Finally we also mark the increase gamut of the impulse responses to the three macroeconomic variables since the 2008 GFC in line with the literature. There are two evident asymmetric response to shocks as in figures 5, and 7. We witness that the 6-day and the 24-day response of the S&P 500 beta in the beginning of COVID-19 are asymmetric, although overall impulse responses show asymmetric response during the whole sample period. The 6-day response is positive while the 24-day response is negative. In addition in panel b of the figures 5 and 7, WTI has negative response to the positive shocks in the macroeconomic variables since GFC. This different compared to the 1990-2008 time period.

As in figures 5 and 6, a shock to VIX generates quick and strong response in the conditional beta and coskewness of both S&P 500 and WTI. On the other side, macroeconomic shocks create stronger 24-day responses on conditional cokurtosis values in figure 6. ADS which is the expectations on business condition index seem to be the macroeconomic variable with the largest impact on all conditional comoments, while economic policy uncertanity (EPU) seem to create stronger effect on the conditional comoments since GFC. This may mark the effect of the sentimental investment in the market.

[8] Note that, in experimenting with longer time intervals for the impulse responses, the responses were negligible to nonexistent above 50 days, which is inline with the nature of our study.

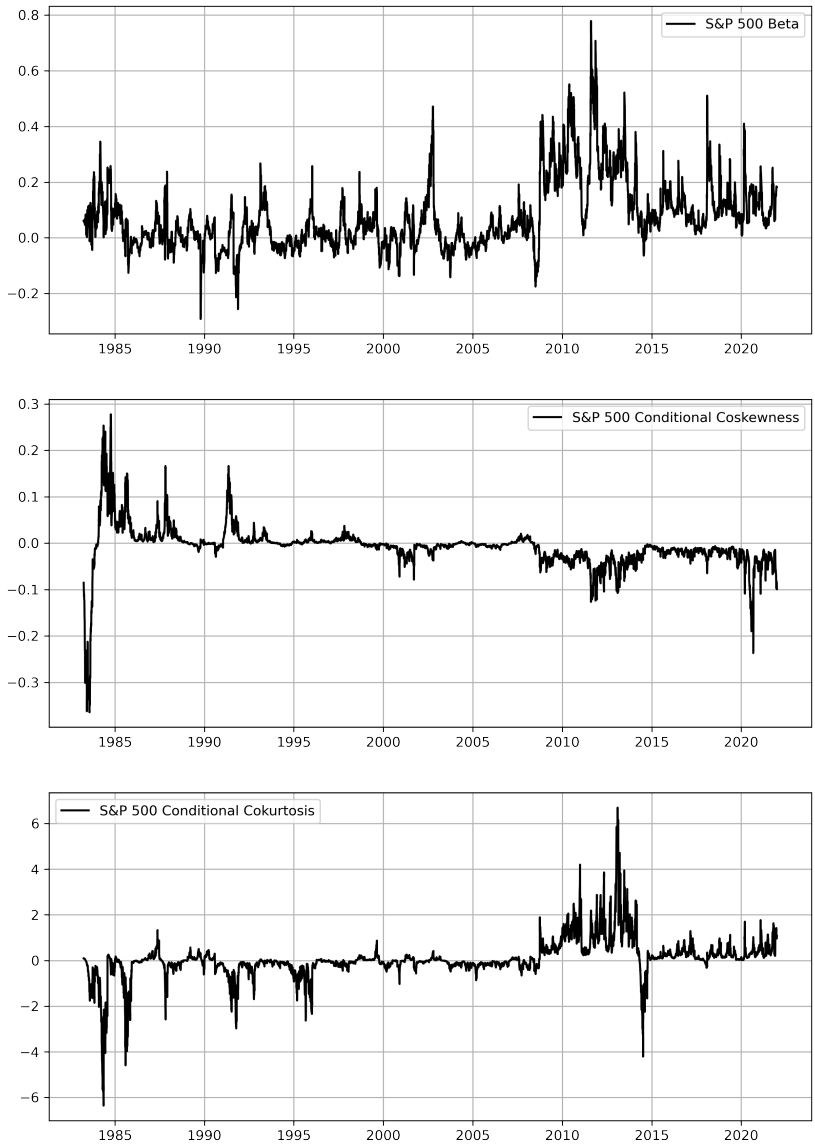


Figure 3: Conditional comoments of S&P 500

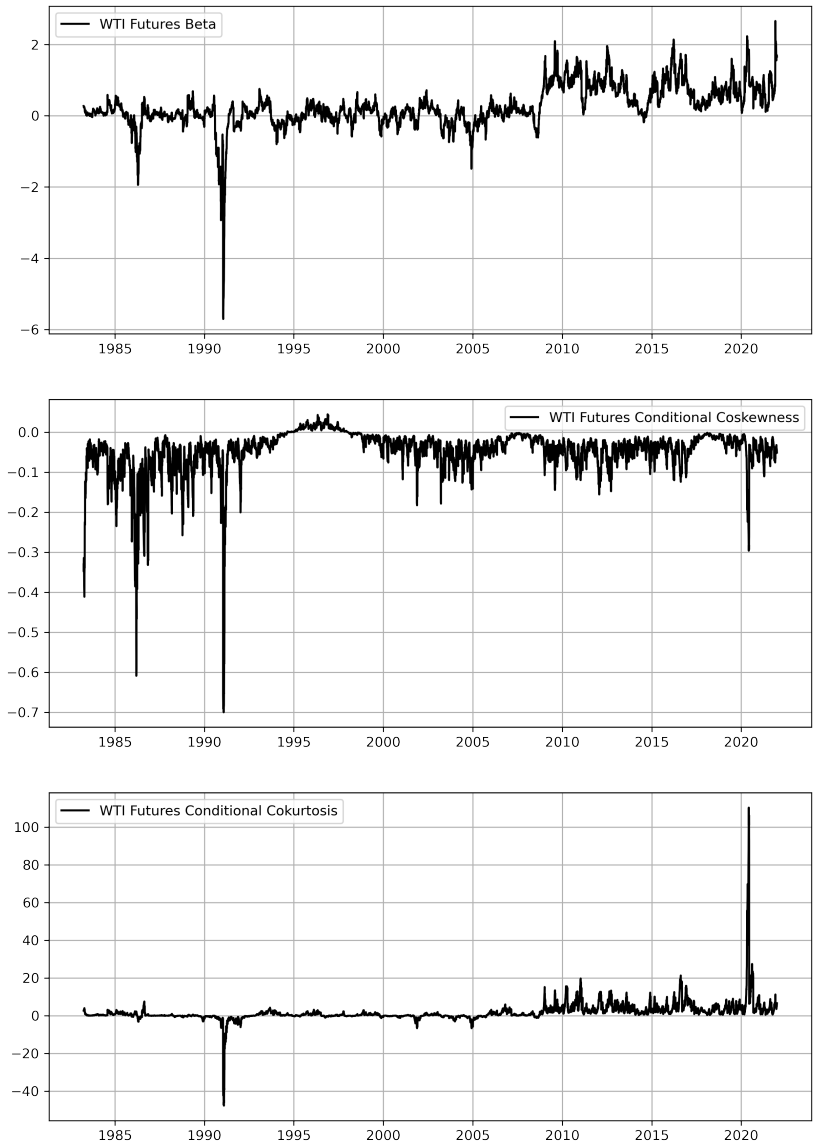
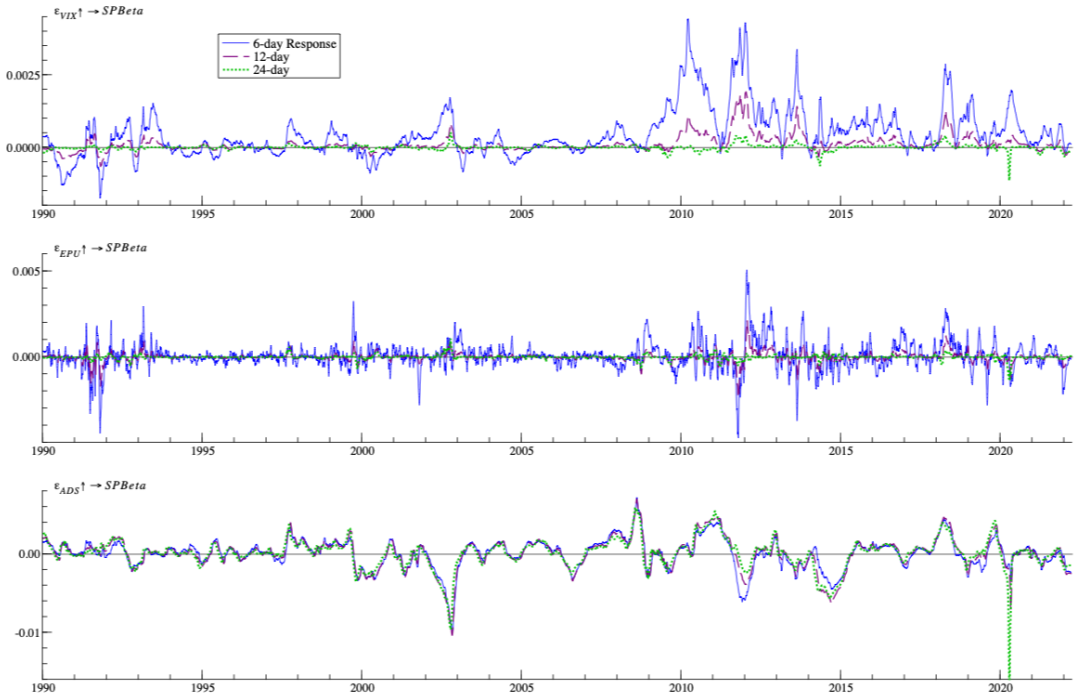
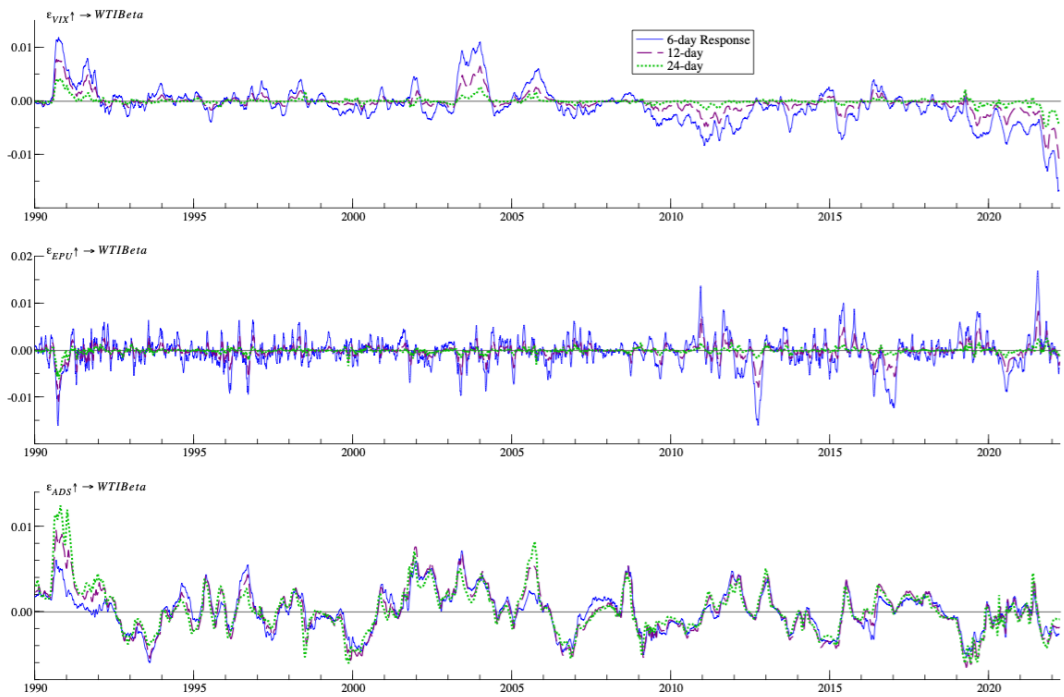


Figure 4: Conditional comoments of WTI Futures

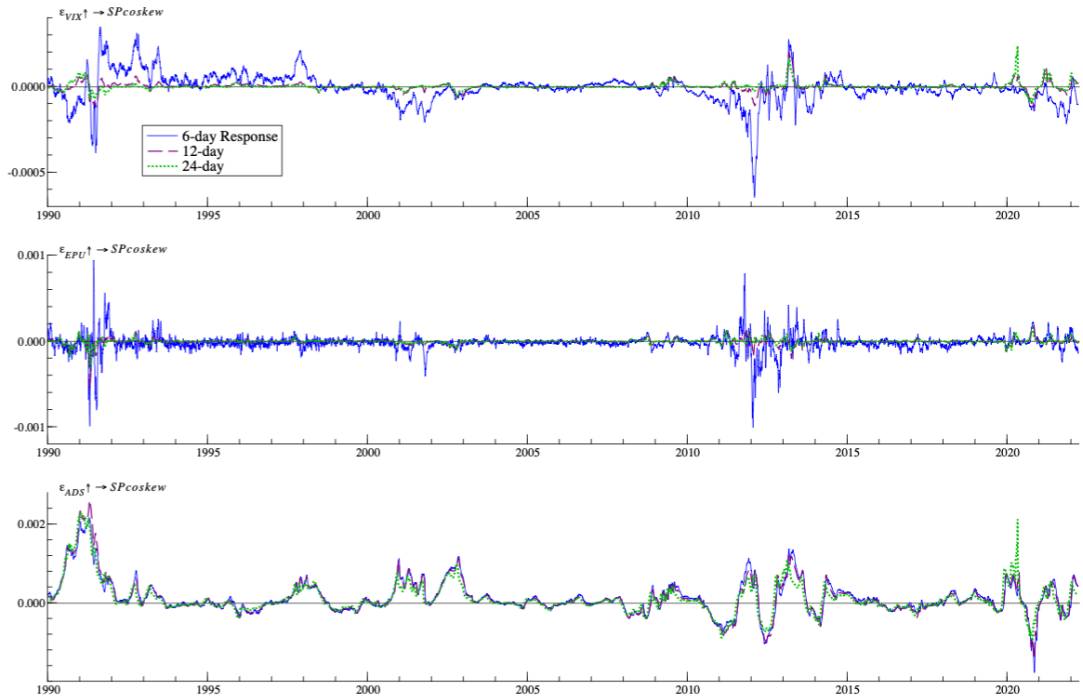


(a) S&P 500 beta impulse responses

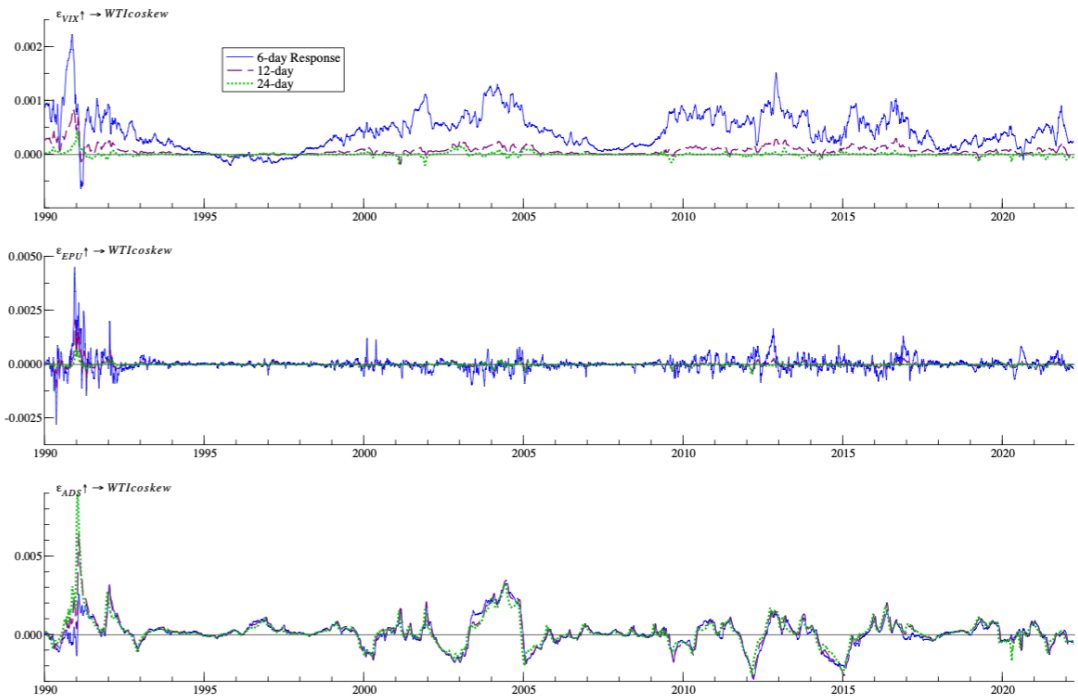


(b) WTI futures beta impulse responses

Figure 5: Time-varying impulse responses for beta Jan 1990 - Dec 2021

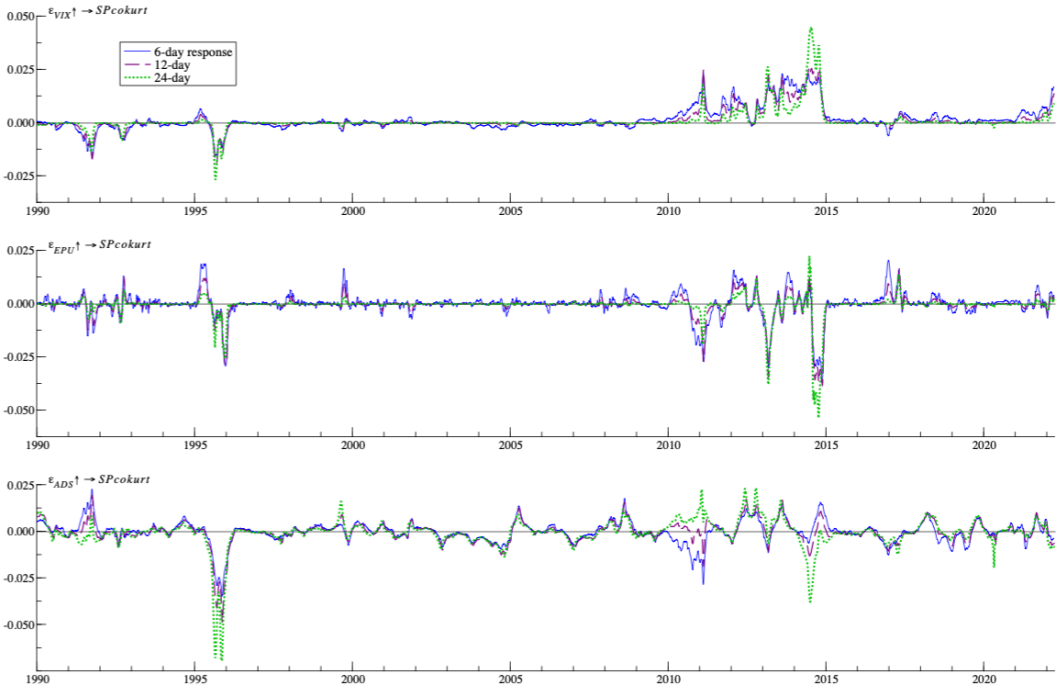


(a) S&P 500 cond. coskewness impulse responses

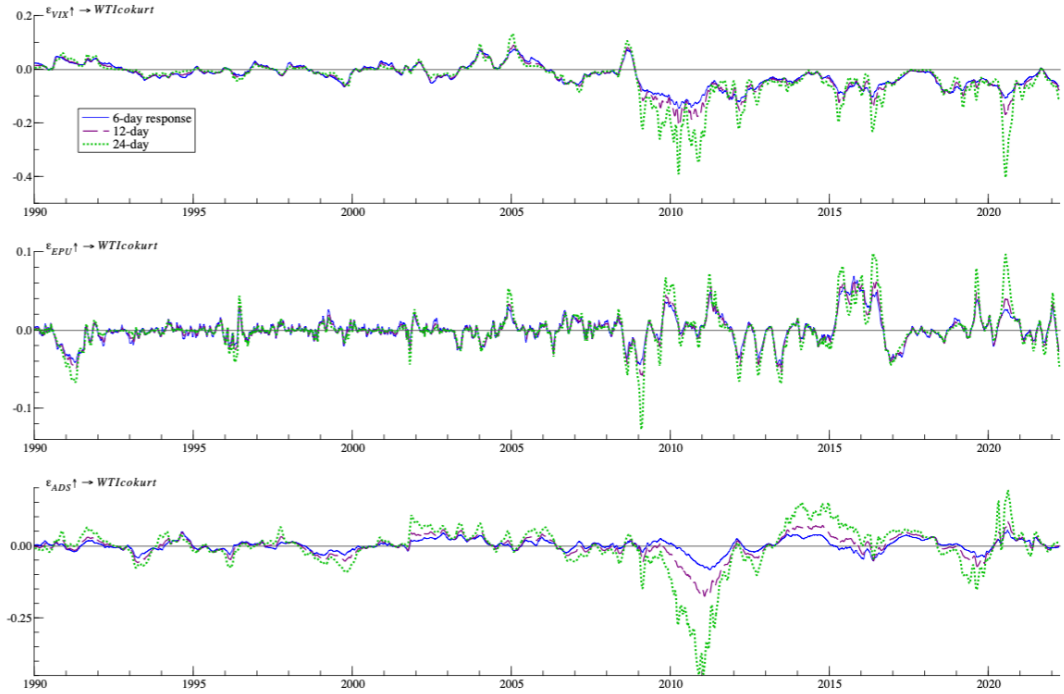


(b) WTI futures cond. coskewness impulse responses

Figure 6: Time-varying impulse responses of cond. coskewness Jan 1990 - Dec 2021



(a) S&P 500 cond. cokurtosis impulse responses



(b) WTI futures cond. cokurtosis impulse responses

Figure 7: Time-varying impulse responses of cond. cokurtosis Jan 1990 - Dec 2021

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