Volatility in Solar Renewable Energy Certificates: Jumps and Fat Tails

Charles F. Mason* Neil A. Wilmot†

December 19, 2021

Abstract

Economies are increasingly adopting renewable energy certificates as a policy mechanism to support the transition away from reliance on fossil fuels. We investigate the price of solar renewable energy certificates (SRECs) in New Jersey, allowing for the potential presence of jumps and time-varying volatility. We find that both features play an important role in the stochastic process describing SREC price returns. We also simulate the implied probability that at least one jump would occur in any given month. These implied probabilities indicate that jumps played a consistently important role in both SRECs and electricity prices. Jumps in SRECs appear to have been particularly noteworthy between late 2011 and early 2013, a period when electricity prices in New Jersey were relatively high. This result hints at the potentially important role of market structure in driving fat tails in price returns.

JEL Codes: L14, L71, O33, O34, Q40
Keywords: Geometric Brownian Motion, Jumps, Fat-tails, Solar prices

*H.A. "Dave" True, Jr. Chair in Petroleum and Natural Gas Economics, Department of Economics, University of Wyoming; bambuzlr@uwyo.edu

†University of Minnesota Duluth; Affiliate Institute on the Environment, University of Minnesota; nwilmot@d.umn.edu

1 Introduction

"Change is inevitable, but transformation is by conscious choice." Amara (2014)

Renewable energy sources form the centerpiece of a strategy aimed at reducing green house gas (GHG) emissions. Indeed, a fundamental transformation of the energy system is likely to be required if mankind is to limit potential future damages from climate change (Nemet, 2019). An important part of this energy transition is likely to entail widespread adoption of renewables in general, and most likely solar power in particular.[1]

Solar power generation produces less GHG emissions than electricity deriving from the combustion of fossil fuels. According to Hosenuzzamana et al. (2015), approximately 3.6×10^4 TW of power is available to us from the sun, while global power consumption in 2018, was 23 TW, a 76% increase since 2000. The Energy Information Administration (*EIA*) forecast that solar power, which only accounted for 3% of US generation in 2020, will grow to 20% by 2050 (U.S. Energy Information Administration, 2021). Indeed, between 2014 and 2020, solar generation from photovoltaic (PV) has increase by 389%, in the US. Such growth has been driven not only by increases in the Utility scale generation (475%), as the small scale generation has also experienced significant growth (271%).[2]

[1] (Nemet, 2019, p. 14) writes: "[m]ajor changes are needed to adapt energy systems in order to ensure that PV [photovoltaic] is a central, rather than peripheral, component of energy systems."

[2] The EIA defines utility-scale solar to be generation of at least 1 MW of capacity, while small-scale solar generation would be less than 1 MW of capacity. For a thorough discussion of the various factors that have contributed to the rise of solar energy see Nemet (2019).

The pace of such an energy transition is a hotly debated topic. When technological improvements are gradual improvements, energy transitions may take "a substantial amount of time." (Sovacool, 2016, p. 205). Large up-front investments are also likely to retard transition (Grubler et al., 2016). On the other hand, elicitation of "expert opinions" often points to a strong tendency to underestimate the pace of technological improvements in solar energy, and thus the speed of transition (Verdolini et al., 2018). Finally, the potential for technological lock-in to older, dirtier technologies – particularly in developing economies – has the potential to retard the energy transition (Fouquet, 2016a).

Numerous policy and regulatory frameworks have been proposed to incentivize the instillation of renewable energy, to achieve long term reductions in carbon emissions. Many US states have introduced a renewable portfolio standard (RPS) to stimulate the production of energy from renewable sources. One policy, that can be used to comply with the RPS mandate, is the use of 'renewable energy certificates' (RECs). These RECs require a tradable certificate be issued to the generator of each MWh of renewable energy, who is then allowed to sell the REC in a market. New Jersey (NJ) was an earlier adopter of this system. Having adopted ambitious future targets for solar generation, NJ instituted a market for solar renewable energy certificates (SRECs), production credits awarded to owners of grid-connected solar PV systems (Coulon et al., 2015).

Like many sources of renewable energy, the lion's share of the cost associated

with a solar project is born up front. As the comments in the preceding paragraph underscore, this raises the potential for slower adoption of solar technology. With large up-front costs, the economic benefits of an investment is substantially driven by the flow of earnings after installation, be they in the form of reduced energy bills (for homeowners), credits from adopting the new technology – such as SRECs – or revenues from electricity sales (Davis and Owens, 2003, p. 1598). This benefit flow must be compared against the (sunk) cost of investment. When either benefits or costs are uncertain, the problem becomes on of "investment under uncertainty" (Dixit and Pindyck, 1994).

The basic idea in this type of problem is that at any point in time the decision-maker may choose to invest immediately or to delay investment. If the payoff to the investment can either increase or decrease, as will naturally occur in the presence of uncertainty, delaying allows the decision-maker to reduce the potential for *ex post* regrets after making an investment that falls in value. In this way, delaying the investment can generate an increase in expected payoff; this increase in value is akin to the value associated with a financial option.[3] A key point is that factors that increase the option value would strengthen the incentive to delay (Dixit and Pindyck, 1994; Kellogg, 2014). These factors include changes in the underlying variance of the source of uncertainty, but they also can include features that contribute to fat tails in the "stochastic process". Such fat tails can arise because of time-varying volatility

[3] Many applications in this genre refer to the investment under uncertainty problem with the moniker "real options".

or the presence of unanticipated dramatic changes, sometimes called "jumps".

Our goal in this paper is to formally assess the empirical importance of jumps and time-varying stochasticity in SREC prices. Using maximum likelihood estimation, we evaluate the percentage change in three key energy prices: SRECs, electricity spot prices and natural gas spot prices. We find that allowing for jumps and time-varying volatility provides statistically important improvements over models which do not account for such features, for each of these energy prices. The preferred econometric specification, which allows for both time-varying volatility and jumps, can then be used to estimate the implied probability that at least one jump would occur over a particular time interval (in our application, one day). Upon constructing these jump probabilities, we find that the estimated jump probability for SREC prices is large at a number of points in time, in particular from late 2011 to early 2013 – a period of relatively high electricity prices in New Jersey. At the same time, the implied probability of jumps in the natural gas prices were much smaller. One important distinction between the markets for SRECs on the one hand and natural gas on the other is the thickness of the market: the volume of trades in SRECs was much smaller, and the market much less mature, than the market for natural gas. This contrast raises the possibility that a thinner market, which one might expect to be less intensely competitive, might be tied to a greater tendency for jumps. Because such a tendency raises the option value of waiting to invest, one conjecture is that tighter markets could be associated with a slower pace of investment – which in this

application suggests an additional factor that could retard the transition to solar power.

2 Literature Review

Increasing interest from state and local policymakers in the development of renewable energy markets has led to the rapid adoption of renewable portfolio standards (RPS). Many of the state RPS laws include a "solar carve-out", which mandates a percentage of power supplied derives from solar generation sources. Hence, RPS creates demand for renewable energy and stimulates investment in renewable energy generation. Krasko and Doris (2013) reports that, between 1996 and 2007, more than half of U.S. states had formalized RPS. The authors examine the use of state policy as a tool to support the development of distributed generation photovoltaic (PV) markets. Using cross sectional data, the authors find evidence suggesting that market supporting policies can be effective at increasing overall PV capacity.

SRECs are financial instruments, created by a state, that are granted to producers of solar power when they generate electricity from solar sources in certain states and under certain conditions (Cohen et al., 2021). SRECs have become an important tool for increasing the proportion of solar energy collected for power generation. However, the nascent nature of this market means that the SREC market experiences significant price volatility. Lee et al. (2017) report eight states in the U.S. have SREC markets, with Massachusetts and New Jersey showing the highest residential average

retail prices (2013). The higher expected price makes residential installation more attractive, allowing residents to both offset home electricity use, and sell excess generation into the SREC market. Yet, the high volatility causes uncertainty of profits and discourages the installation of solar PV systems. Using county level data for 13 Northeastern states, Crago and Chernyakhovskiy (2017) examine the impact of policy incentives for residential PV capacity. These authors report that, while rebates are a highly significant predictor of solar capacity additions, SREC prices were not significant. They further argue that the lack of significance may be due to the volatility in the prices during the period studied. Under North Carolina's RPS compliance market, installation of solar power has only enabled a few large solar power producers to compete with utility companies to finance, install, and operate solar generating systems (Gaul and Carley, 2012).

The consequences of compressed investment from heightened price volatility is a well documented in the energy literature. Pindyck (2004) examines the behavior of volatility in both the natural gas and oil markets, arguing that volatility alters the incentives to invest in production facility, storage, and transport infrastructure. Indeed, volatility is a key element for the pricing of derivatives, hedging strategies and the decision to invest in physical capital (real options). Relatedly, Efimova and Serletis (2014) show that GARCH models explain price volatility for crude oil, natural gas and electricity. Mason and Wilmot (2016a) show that delaying investment in physical assets could occur because of discontinuities – or "jumps" – in the relevant

market price. Similarly, in an analysis of the market for renewable identification numbers (RINs), Mason and Wilmot (2016b) show that a stochastic model which includes jumps provides a compelling explanation of the data. Their results indicate the statistical importance of incorporating jumps in a model of price returns, and the delaying effect such jumps have on undertaking capital investment.

The empirical importance of jumps has been documented for a variety of energy markets, including oil (Askari and Krichene, 2008; Gronwald, 2012; Postali and Picchetti, 2006; Wilmot and Mason, 2013), natural gas (Benth et al., 2008; Mason and Wilmot, 2016a), carbon permits in the European Union trading market (Alberola et al., 2008; Chevallier and Sévi, 2014; Daskalakis et al., 2009; Hammoudeh et al., 2014), and coal (Wilmot, 2016; Xiaoming et al., 2012).[4] A natural consequence of the jumps in energy commodity prices is that electricity spot prices can also exhibit jumps (Benth et al., 2008; Huisman and Mahieu, 2003). The jumps in these energy prices can be large: Huisman and Mahieu (2003) show that daily jumps in electricity prices can be on the order of 30%. Similarly, Thompson et al. (2009, p. 227) observe "high [natural gas] price spikes far outside normal seasonal equilibrium levels;" and Chen and Forsyth (2010, p. 359) argue it "is not uncommon to see spot gas price

[4] Indeed, Chevallier and Ielpo (2014) argue that many energy prices are subject to these effects. These energy markets can sometimes be correlated Gonzalez-Pedraz et al. (2014); Asche et al. (2006) and Panagiotidis and Rutledge (2007) demonstrate a link between UK natural gas prices and European crude prices. On the other hand, (Brown and Yücel, 2008, p. 48) argue that there has been a fundamental change in the relationship between oil and gas prices after 2000. And while the data from 2000 – 2006 is reflective of emerging technological trends, it predates the major burst in gas production that followed the widespread application of fracking. With prices subject to jumps, it is not unexpected that related derivatives would also exhibit jumps – for example, the convenience yield associated with holding crude oil stockpiles (Mason and Wilmot, 2020).

jumps ... as much as 20% in a single day."

Similar to this earlier literature, there is evidence that the SREC market in NJ has exhibited volatile price dynamics (Coulon et al., 2015); this has the potential to reduce investments in solar power generation due to the increased risk from large price swings. These authors propose a stochastic model to examine the behavior of SREC market prices, which includes a feedback mechanism for power generation decisions. Utilizing monthly data, over the period July 2008 through March 2014, they model the historical dynamics of SREC prices. The model requires numerical solution algorithm, which the authors argue must be carefully chosen, as the historical SREC price behavior is extremely challenging to capture or understand in a classical econometric price model.

Understanding the nature of these price movements has important implications for large-scale investment decisions, particularly when these decisions entail up-front costs that are costly (or impossible) to reverse. Here too there is a long tradition in the literature of analyzing investment decisions when key ingredients, such as underlying prices, are stochastic. Much of this literature follows the seminal work of Dixit and Pindyck (1994). A key insight here is that there is a value to waiting – reflecting the potential for the gain from investing to increase over time; this value is referred to as the "option value" of delaying investment. In general, the larger is the option value from delaying, the later will the investment be undertaken. Of direct relevance to our analysis, Torani et al. (2016) evaluate a model of investment in solar projects under

uncertainty. Their model assumes solar installation prices follow geometric Brownian motion, though they allow for neither GARCH-style volatility nor jumps in prices.[5] We do allow for both GARCH and jumps in our model of SREC prices; we argue below that both aspects are statistically important.

These results have clear implications for a broad range of decisions involving renewable energy, from smaller decisions (such as a homeowner's decision to install rooftop solar) to larger decisions (such as a utility's decision to install a substantial solar farm). The implications are particularly significant for very large decisions.[6] There is a robust literature that has studied problems of investment under uncertainty; many of these papers study applications to energy and climate economics. A number of authors have studied applications to non-renewable energy resources, such as oil or gas, often with a focus on production from a single well. A representative sample of these papers includes Almansour and Insley (2017); Conrad and Kotani (2005); Gonzalo Cortazar and Jaime Casassus (1998); Davis and Lund (2018); Insley (2017); Kellogg (2014); Mason (2001). Applications that corroborate the empirical validity of the real options framework include Muehlenbachs (2015) for crude oil in Alberta and Paddock et al. (1988) for offshore oil and gas leases. The broad consensus from this literature is that the development of energy resources can often be characterized in a fashion that is consistent with real option theory. In particular, the

[5] They also argue that electricity price uncertainty plays an important role in impacting the volatility of solar markets.

[6] For example, the cost of a recently cited wind farm in southern Wyoming is expected to exceed one billion US dollars.

notion of the option value of waiting, which is central to our discussion, is empirically relevant in these applications.

3 Methods and Materials

3.1 Econometric Methods

To present the stochastic process under investigation, let P_t denote the price of *SERC* at time t, which follows a geometric Browniam motion *(GBM)* process. Define α as the trend, and σ as the variance, so that

$$dP_t = \alpha P_t d_t + \sigma P_t dz. \tag{1}$$

In Eq. 1, dz represents an increment of a Wiener process, $dz = \xi\sqrt{dt}$, where ξ has zero mean and a standard deviation equal to 1. (Dixit and Pindyck (1994)). Let x_t denote the natural logarithm of the ratio of price in period t to the price in period $t-1$, $x_t = ln(P_t) - ln(P_{t-1})$. If P_t follows a *GBM* process then x_t is normally distributed with variance σ^2 and mean $\mu = \alpha - \sigma^2/2$. This gives the pure diffusion *(PD)* model

$$x_t = \mu + \sigma z_t \tag{2}$$

The term z_t in Eq. 2 is an identically and independently distributed (i.i.d.) random variable with mean zero and variance one.

As we observed in Section 2 above, many authors have argued that continuous stochastic processes, such as *GBM*, are insufficient for explaining discontinuous movements, or "jumps", in prices. To introduce jumps into the above model, we adapt the methodology in Merton (1976) – wherein returns are composed of two types of changes. The first, 'normal' fluctuations, are represented through the geometric Brownian motion process, while the second – 'abnormal' shocks due to the arrival of new information – are modeled through a discontinuous process. The discontinuities are described by the Poisson distribution governing the number of discrete-valued events, $\eta_t \in \{0, 1, 2, ...\}$, that occur over the interval $(t-1, t)$,

$$P(N_t = j) = \frac{\exp(-\lambda)\lambda^j}{j!} \tag{3}$$

Eq. 3 is characterized by the jump intensity, λ, which describes the mean number of 'events' per unit of time and is expressed as

$$dn_t = \begin{cases} 0 & \text{with probability } 1 - \lambda dt \\ 1 & \text{with probability } \lambda dt \end{cases} \tag{4}$$

As in Askari and Krichene (2008), when abnormal information arrives at time t, prices jump from P_{t-} (the limit as the time index tends towards t from left) to $P_t = \exp(J_t) P_{t-}$. Accordingly, J_t measures the percentage change in price. The

resultant stochastic process for the random variable P_t may then be written as

$$\frac{dP_t}{P_t} = \alpha dt + \sigma dz_t + (\exp(J_t) - 1)\, dn_t, \tag{5}$$

where dz_t has the same properties assumed in equation (1) and dn_t is the independent Poisson process described in equation (4). Together the terms dz_t and dn_t make up the instantaneous component of the unanticipated return. It is natural to assume these terms are independent, since the first component reflects ordinary movements in price while the second component reflects unusual changes in price. The size of the jump, $Y_{t,k}$, is itself a random variable; we assume it is normally distributed with mean θ and variance δ^2, and that it is independent of the distribution for the arrival of a jump. The jump component affecting returns between time t and time *t+1* is then

$$J_t = \sum_{k=0}^{n_t} Y_{t,k}. \tag{6}$$

Thus, the mixed jump-diffusion (JD) process for the log-price returns can be described by

$$x_t = \mu + \sigma z_t + J_t. \tag{7}$$

The probability density function governing x can be derived by applying Bayes' law (Chan and Maheu, 2002; Maheu and McCurdy, 2004). To this end let $f(x_t|n_t = j, \Omega_{t-1})$ denote the conditional density of returns if j jumps have occurred and given the available information Ω_{t-1}. Based on Bayes rule, when x_t is observed, the posterior

probability that j jumps will occur at time t is

$$P(n_t = j|\Omega_{t-1}) = \frac{f(x_t|n_t = j, \Omega_{t-1})P(n_t = j|\Omega_{t-1})}{P(x_t|\Omega_{t-1})}. \tag{8}$$

Then, assuming that the conditional density of x_t is normally distributed, and using equation (3) to describe the probability that j jumps occur, we obtain:

$$f(x_t|n_t = j, \Omega_{t-1}) = \frac{1}{\sqrt{2\pi(\sigma^2 + j\delta^2)}} \exp(-\frac{(x_t - \mu - \theta j + \theta\lambda)^2}{2(\sigma^2 + j\delta^2)}). \tag{9}$$

Finally, integrating out the discrete valued number of jumps yields an expression for the conditional density in terms of observable variables:

$$P(x_t|\Omega_{t-1}) = \sum_{j=0}^{\infty} f(x_t|n_t = j, \Omega_{t-1})P(n_t = j|\Omega_{t-1}). \tag{10}$$

An alternative explanation for the "fat tails" that are often observed in commodity price data is that P_t is subject to time-varying volatility. An example of such a phenomenon is the "generalized autoregressive conditional heteroskedastic" (GARCH) framework. Adapting the pure diffusion model to allow for this form of time-varying volatility gives the GARCH–diffusion (GPD) process:

$$x_t = \mu + \sqrt{h_t} z_t, \tag{11}$$

where the conditional variance, h_t is described by the process

$$h_t = \kappa + \alpha_1 (x_{t-1} - \mu)^2 + \beta_1 h_{t-1}. \tag{12}$$

Note that when $h_t = \sigma^2$ the GARCH–diffusion model *(GPD)* reduces to pure diffusion model *(PD)*. On the other hand, when $\kappa > 0$ and $\alpha_1 + \beta_1 < 1$, the unconditional variance of the volatility of the process exists and equals $\frac{\kappa}{1-\alpha_1-\beta_1}$. The process described in equations (11) – (12) is characterized by four parameters, μ, κ, α_1 and β_1. There is a general consensus in the literature is that a GARCH model with a limited number of terms performs reasonably well (Akgiray, 1989; Hansen and Lunde, 2005; Sadorsky, 2006), and so the focus of the analysis is restricted to this more parsimonious representation.

Allowing for jump discontinuities would result in the GARCH(1,1) jump-diffusion (*GJD*) process:

$$x_t = \mu + \sqrt{h_t} z_t + J_t, \tag{13}$$

where h_t is described by equation (12). Duan (1997) shows that the diffusion limit of a large class of GARCH(1,1) models contain many diffusion processes allowing the approximation of stochastic volatility models by the GARCH process.

We evaluate the four models using maximum likelihood estimation methods, which are known to provide consistent and invariant estimates, with asymptotically normal distributions of the parameters. To this end, we note that the parameters of our

four candidate models – *PD, JD, GPD, GJD* – may be nested into the general log-likelihood function

$$L\left(\phi, x_t\right) = -T\lambda - \frac{T}{2}\ln\left(2\pi\right) + \sum_{t=1}^{T}\ln\left[\sum_{n=0}^{\infty}\frac{\lambda^n}{n!}\frac{1}{\sqrt{h_t + n\delta^2}}\exp\left(\frac{-\left(x_t - \mu - n\theta\right)}{2\left(h_t + n\delta^2\right)}\right)\right], \tag{14}$$

where n indexes the number of jumps, combined with the description of h_t given in equation (12).[7] In this framework, the GPD model corresponds to the parameter restriction $\lambda = \theta = \delta = 0$; the JD model corresponds to the restriction $\alpha_1 = \beta_1 = 0$; and the PD model corresponds to the restriction $\alpha_1 = \beta_1 = \lambda = \theta = \delta = 0$.

Comparing any pair of potential models can thus be framed as a test of an appropriate parameter restriction. For example, the comparison of the PD and GPD models is conducted by testing the parameter restriction $\alpha_1 = \beta_1 = 0$; the comparison of the PD and JD models is conducted by testing the parameter restriction $\lambda = \theta = \delta = 0$. The empirical validity of the parameter restriction of interest can be evaluated by use of the likelihood ratio test. This approach compares the likelihood function under a particular restriction, $L(\phi^R;x)$, to that of the unrestricted or less restricted likelihood function, $L\left(\hat{\phi};x\right)$. Under the null hypothesis that the restriction is empirically valid, the decrease in the likelihood function associated with the restriction will be small. Such an approach can be used to make pairwise-comparisons between a more general model and a more restricted model. The test statistic is the

[7] To faciliate estimation based on eq. (14), the number of jumps was truncated at 10 (Ball and Torous, 1985). Allowing more jumps did not significantly change the point estimates.

log-likelihood ratio

$$LR = 2[L(\hat{\phi}; x) - L(\phi^R; x)];$$

under the null hypothesis this statistic will be distributed as a Chi-square random variable with m degrees of freedom, where m is the number of parameter restrictions.

3.2 Data

We use daily data used in our analysis. SREC prices were obtained from the New Jersey Clean Energy program website.[8] Natural gas prices are based on the Henry Hub spot price.[9] For electricity, we used the PJM Western hub price[10], as this range includes New Jersey.

Table 1 provides summary statistics for the log-returns, calculated as $r_t = ln(x_t) - ln(x_{t-1})$, for each of the prices returns series. The statistics indicate a significant amount of variation among the three variables, though SERC returns exhibit the lowest relative level of dispersion. Evidence of asymmetry in the distribution of returns is also highest for SERC returns, while electricity prices exhibit negative skewness. The "fat tails" are evident from the large values of kurtosis. The normality hypothesis of the Kolmogorov-Smirnov test is rejected, for each time-series. Lastly, Table 1 contains the results of the modified ADF test of nonstationarity. In each of the log return series the null hypothesis of a unit root is rejected, in favor of

[8] Accessed at https://njcleanenergy.com/renewable-energy/project-activity-reports/srec-pricing/srec-pricing-archive

[9] https://www.eia.gov/dnav/ng/ng_pri_fut_s1_d.htm

[10] https://www.eia.gov/electricity/wholesale/

stationarity.[11]

These results are corroborated by Figures 1 and 2. Figure 1 displays the log returns for the three energy series. Notably, large sudden changes in the returns are evident, and appear with less frequency for natural gas returns, though on a larger scale for SREC solar price returns. Also, the large changes in SREC solar price returns are evident, and the positive shocks appear much larger in amplitude than the negative changes. Figure 2 displays "quantile-quantile" plots, for the three return series. This plot compares the quantiles of the empirical distribution (measured on the y-axis) against the quantiles of a theoretical normal distribution (measured on the x-axis). If the empirical distribution is close to a normal distribution, the relation will be well described by a straight line. As demonstrated in the figure, there are significant departures from a linear relation, indicating that the date is not well-described by a normal distribution. Indeed, the significant departures observed in along the ends of the plots are indicative of significant leptokurtosis and cast doubt on the assumptions of an underlying GBM process.

[11] While not reported in the table, the SREC solar credit, natural gas, and electricity prices are nonstationary (in levels), and thus first differencing (e.g. log returns) renders the series stationary. The findings are supported by traditional unit root tests, the results of which are available upon request.

4 Results

4.1 Maximum Likelihood Estimation

Using maximum likelihood methods, we estimate the parameters for the four stochastic processes (*PD, JD, GPD, GJD*), using the daily returns series. The results of the estimation are presented in Table 2, while the likelihood ratio test statistics, for the pairwise comparison of the four models, are presented in Table 3.

Examining the pure diffusion results, the estimated drift parameter, μ, is positively and statistically significant for SREC returns, though not statistically significant for either natural gas or electricity prices.[12] For each series, the estimated instantaneous rate of variance, σ, is statistically significant. This same pattern holds when examining the drift and variance of the JD models, though allowing for jumps markedly reduces the estimated instantaneous rate of variance. Across all three series, the jump intensity, λ, is statistically significant in each case. The estimated value of λ is largest for the SREC series; this estimate suggests jumps occur quite frequently for SREC returns – roughly two out of every three days. Finally, the estimated mean jump size, θ, is positive when the estimate is significant.

Comparing the results of the PD and GPD models, we see that allowing for GARCH also improved the models predictive power: the log-likelihood function shows improvement, in each of the three series. The estimated coefficients associated with

[12] To improve convergence of the estimation procedure, we rescaled the price return series for natural gas and electricity by multiplying by 100; the SREC returns were not rescaled.

the GARCH model, κ, α_1 and β_1, are statistically significant across each of the three series. Additionally, all three series demonstrate persistent volatility, with $\alpha_1+\beta_1$ close to 1. Notably, the magnitude of the estimated value of κ is large for electricity price returns – pointing to large overall variation in that series. Importantly, our results indicate that combining GARCH with the jump model (GJD) yield the best results, demonstrating the importance of including jumps. We note that the probability of a jump occurring has fallen across all three series, while the GARCH terms remain statistically significant. Evidently, the inclusion of GARCH captures some of the estimated variance originally interpreted as jumps in the JD model, it does not render the jumps irrelevant. Indeed, based on the decreased values of λ, and notable increases in θ, the average jump size, one conceivable theory is that the small perturbations are captured by the GARCH model while the larger discontinuous moves are represented as jumps.

4.2 Likelihood Ratio Test Results

Results of the pairwise likelihood ratio tests are presented in Table 3. Each entry in the table is a test of the form $LR_{x,y}$, where the null hypothesis is that the appropriate stochastic process describing the data is x and the alternative hypothesis is that the appropriate stochastic process describing the data is y. The probability that the null hypothesis is preferred to the alternative, based upon the Chi-squared distribution, is presented in the parentheses below each test statistics. The results indicate that

the null hypothesis is rejected at the 1% level of significance in every case. Such a result is interpreted as demonstrating the statistically important gain in predictive power associated with adopting the added complexity associated with moving to specification y, from x. Thus, for example, including GARCH improves upon the JD model, while folding in jumps improves upon the GARCH model. Importantly, the preferred specification allows from both jumps and time-varying volatility (GJD), for each of the three series.

4.3 Implied Probabilities

Figure 3 gives a visual representation of the implied probabilities of at least one jump in the log price returns, occurring on each week, based on the JD model. The first panel shows the probabilities for the SREC solar credit returns. Given the frequency and persistence of implied probabilities that are well above 0.5, it is apparent that jumps play an important role in both SREC and electricity markets. We note too that the implied jumps close to 1 in the SREC markets are particularly common in 2012; this is intriguing as electricity prices in New Jersey were particularly large during this period. In contrast to the other two markets, the implied jump probabilities in the natural gas are smaller, and less frequently close to 1. One possible explanation for this distinction is that natural gas markets are thicker than the other markets. In addition, the SREC market is noticeably less mature, having only come into fruition in the last fifteen years; by contrast, natural gas markets have been functioning for

many years. One conjecture is that market thickness as well as maturity might create a form of insulation from large transitory events – thereby rendering jumps more important in younger or thinner markets, such as the SREC market.

5 Discussion

In this study we investigated the price of solar renewable energy certificates (SRECs) in New Jersey – production credits awarded to owners of grid-connected solar PV systems – allowing for the potential presence of jumps and time-varying volatility. Our results point to the importance of both of these features in modeling SREC price returns. In addition to providing a statistically important improvement in explanatory power of the empirical model, our results allow us to simulate the implied probability that at least one jump would occur in any given month. Following this approach, we developed implied jump probabilities for price returns for SRECs, as well as natural gas and electricity price returns. Our results point to the consistently important role of jumps in both SRECs and electricity prices, but less so for natural gas prices. Jumps in SRECs appear to have been of particular significance in the period between late 2011 and early 2013, a period when electricity prices in New Jersey were particularly high – presumably providing heightened incentives for homeowners to invest in rooftop solar. Two important factors that distinguish SREC and natural gas markets are the relative thickness of the market and its maturity. Natural gas markets are commonly thought to be quite thick and mature, whereas SREC markets

have been in place for a much shorter period, and there are far fewer suppliers of solar projects. This points to the potentially important role of market structure in driving fat tails in price returns.[13]

The rapid growth in solar PV capacity has been driven by the falling price of installing PV and the implementation of public policies (Nemet, 2019). Even so, rooftop solar panel installations are a non-trivial expense to the homeowner; Dastrup et al. (2012) estimate the cost to be on the order of 3-4 % of the home value. Perhaps because of this feature, Mamkhezri et al. (2020, p. 178) have argued that "there is a diminishing return in support for [r]ooftop solar." As we noted in section 1, the presence of uncertainty can retard investment, particularly in projects with largely irreversible up-front investments such as rooftop solar installations. The obvious alternative would be a large-scale solar farm. But the private incentive to undertake such a large investment depends importantly on the option value of waiting to invest. This delaying effect is larger the larger is the variation in the stochastic element of concern (here, SREC prices). Importantly, the presence of jumps mimics an increase in variability (Mason and Wilmot, 2021). As such, our finding that both jumps and time-varying price volatility provide a statistically important explanation of the pattern in SREC prices is of some concern, as there is reason to fear these effects will increase the option value of waiting – and thereby retard the rate of investment in so-

[13] Of course, market structure can exert an important influence on price levels – either by allowing sellers to charge higher prices or by allowing sellers to capitalize on cost reductions, for example through scale economies (Gillingham et al., 2016). Our point is that more concentrated markets may be more susceptible to the influence of unexpected events that could cause dramatic changes in prices.

lar energy. The upshot is that development of solar farms and similar investments are likely to occur at a somewhat slower pace than one might prefer – slowing the energy transition. This effect is reminiscent of, but operates through a different channel to, the potential delay in investment that arises when learning-by-doing effects are important (van Benthem et al., 2008). When learning-by-doing effects are present, private decision-makers undervalue the potential future benefits associated with present actions, *i.e.* they overweight current effects. When investments are delayed the option value of waiting, decision-makers see increased present gains in part because of the desire to capture potential future gains that might arise if jumps occur. While the implications for current investment are similar to learning models, the mechanism differs.

On a more optimistic note, there are signs that energy investments in developing countries are being directed into renewables: a recent World Bank report argues that a large share of new electricity-generation projects in developing countries are renewable-energy, with a focus on solar (World Bank, 2021). On the other hand, to the extent that developing countries are undertaking energy investments (as opposed to external players making the investments), there are concerns that these investments are often directed into technologies that are too carbon-intensive to meet commonly articulated climate goals (Ball et al., 2021). Moreover, there are possible "resource curse" concerns associated with large solar projects, particularly in developing countries with large areas amenable to solar development (O'Sullivan et al., 2017, p. 18).

To the extent that expanding solar deployment in developing countries entails large up-front investments, or that there is a link between market power and energy system lock-ins (Fouquet, 2016b), these factors can reinforce the effects that create an option value of delaying investment. If so, a concern is that jumps could exacerbate this retardation in the energy transition in developing countries.

References

Akgiray, V. (1989). Conditional Heteroskedasticity in Time Series of Stock Returns: Evidence and Forecasts, *Journal of Business* **62**: 55 – 80.

Alberola, E., Chevallier, J. and Chèze, B. (2008). Price Drivers and Structural Breaks in European Carbon Prices 2005 – 2007, *Energy Policy* **36**: 787 – 797.

Almansour, A. and Insley, M. (2017). The impact of stochastic extraction cost on the value of an exhaustible resource: An application to the Alberta oil sands, *The Energy Journal* **37**: 61 – 88.

Amara, H. (2014). *Warrior Goddess Training: Become the Woman You Are Meant to Be*, Hierophant Publishing.

Asche, F., Osmundsen, P. and Sandsmark, M. (2006). The UK market for natural gas, oil and electricity: Are the prices decoupled?, *The Energy Journal* **27**: 27–40.

Askari, H. and Krichene, N. (2008). Oil Price Dynamics (2002 – 2006), *Energy Economics* **30**: 2134 – 2153.

Ball, C. A. and Torous, W. N. (1985). On Jumps in Common Stock Prices and Their Impact on Call Option Pricing, *The Journal of Finance* **40**: 155–173.

Ball, J., Pastor, A. O., Liou, D. and Dickey, E. (2021). Hot Money: Illuminating the Financing of High-Carbon Infrastructure in the Developing World, **24**. Article 103358.

Benth, E. F., Benth, J. and Koekbakker, S. (2008). *Stochastic Modelling of Electricity and Related Markets*, World Scientific, River Edge, NJ.

Brown, S. P. A. and Yücel, M. K. (2008). What drives natural gas prices?, *The Energy Journal* **29**: 45 – 60.

Chan, W. H. and Maheu, J. M. (2002). Conditional Jump Dynamics in Stock Market Returns, *Journal of Business and Economic Studies* **20**: 377 – 389.

Chen, Z. and Forsyth, P. A. (2010). Implications of a Regime-Switching Model on Natural Gas Storage Valuation and Optimal Operation, *Quantitative Finance* **10**: 159 – 176.

Chevallier, J. and Ielpo, F. (2014). Twenty years of jumps in commodity markets, *International Review of Applied Economics* **28**(1): 64–82.

Chevallier, J. and Sévi (2014). On the stochastic properties of carbon futures prices, *Environmental and Resource Economics* **58**: 127 – 153.

Cohen, J. J., Elbakidze, L. and Jackson, R. (2021). "interstate protectionism: The case of solar renewable energy credits", *American Journal of Agricultural Economics* **forthcoming**: 1 – 22.

Conrad, J. M. and Kotani, K. (2005). When to drill? trigger prices for the Arctic National Wildlife Refuge, *Resource and Energy Economics* **27**: 273 – 286.

Coulon, M., Khazaei, J. and Powell, W. B. (2015). Smart-srec: A stochastic model of the new jersey solar renewable energy certificate market, *Journal of Environmental Economics and Management* **73**: 13–31.

Crago, C. L. and Chernyakhovskiy, I. (2017). Are Policy Incentives for Solar Power Effective? Evidence From Residential Installations in the Northeast, *Journal of Environmental Economics and Management* **81**: 132–151.

Daskalakis, G., Psychoyios, D. and Markellos, R. N. (2009). Modeling CO2 emission allowance prices and derivatives: Evidence from the European trading scheme, *Journal of Banking & Finance* **33**: 1230 – 1241.

Dastrup, S. R., Graff Zivin, J., Costa, D. L. and Kahn, M. E. (2012). Understanding the Solar HomePrice Premium: Electricity Generation and "Green" Social Status, *European Economic Review* **56**: 961 – 973.

Davis, G. A. and Lund, D. (2018). Taxation and Investment Decisions in Petroleum, *The Energy Journal* **39**: 189 – 208.

Davis, G. A. and Owens, B. (2003). Optimizing the Level of Renewable Electric R&D Expenditures Using Real Options Analysis, *Energy Policy* **31**: 189 – 208.

Dixit, A. and Pindyck, R. S. (1994). *Investment under Uncertainty*, Princton University Press, Princton, New Jersey.

Duan, J. C. . (1997). Augmented GARCH(p,q) process and its diffusion limit, *Journal of Econometrics* **79**: 97–127.

Efimova, O. and Serletis, A. (2014). Energy Markets Volatility Modelling using GARCH, *Energy Economics* **43**: 264 – 273.

Elliot, G., Rothenberg, T. J. and Stock, J. H. (1996). Efficient Test for an Autoregressive Unit Root, *Econometrica* **64**: 813 – 836.

Fouquet, R. (2016a). Lessons from energy history for climate policy: Technological change, demand and economic development, *Energy Research & Social Science* **22**: 79 – 93.

Fouquet, R. (2016b). Path Dependence in Energy Systems and Economic Development, *Nature Energy* **1**: 1 – 5. article 16098.

Gaul, C. and Carley, S. (2012). "solar set asides and renewable electricity certificates: Early lessons from north carolina's experience with its renewable portfolio standard", *Energy Policy* **48**: 460 – 469.

Gillingham, K., Deng, H., Wiser, R., Darghouth, N., Nemet, G., Barbose, G., Rai, V. and Dong, C. (2016). Deconstructing Solar Photovoltaic Pricing, *The Energy Journal* **37**: 231 – 250.

Gonzalez-Pedraz, C., Moreno, M. and Peña, J. I. (2014). Tail risk in energy portfolios, *Energy Economics* **46**: 422 – 434.

Gonzalo Cortazar and Jaime Casassus (1998). Optimal timing of a mine expansion: Implementing a real options model, *The Quarterly Review of Economics and Finance* **38**: 755 – 769.

Gronwald, M. (2012). A Characterization of Oil Price Behavior – Evidence from Jump Models, *Energy Economics* **34**: 1310 – 1317.

Grubler, A., Wilson, C. and Nemet, G. (2016). Apples, Oranges, and Consistent Comparisons of the Temporal Dynamics of Energy Transitions, *Energy Research & Social Science* **22**: 18 – 25.

Hammoudeh, S., Nguyen, D. K. and Sousa, R. M. (2014). What explain the short-term dynamics of the prices of CO2 emissions?, *Energy Economics* **46**: 122 – 135.

Hansen, P. R. and Lunde, A. (2005). A Forecast Comparison of Volatility Models: Does Anything Beat a GARCH (1,1)?, *Journal of Applied Econometrics* **20**: 873 – 889.

Hosenuzzamana, M., Rahimab, N., Selvaraja, J., Hasanuzzamana, M., Maleka, A. and Nahara, A. (2015). Global Prospects, Progress, Policies, and Environmental Impact of Solar Photovoltaic Power Generation, *Renewable and Sustainable Energy Reviews* **41**: 284 – 297.

Huisman, R. and Mahieu, R. (2003). Regime jumps in electricity prices, *Energy Economics* **25**: 425 – 434.

Insley, M. (2017). Resource extraction with a carbon tax and regime switching prices: Exercising your options, *Energy Economics* **67**: 1 – 16.

Kellogg, R. (2014). The effect of uncertainty on investment: Evidence from texas oil drilling, *American Economic Review* **104**: 1698 – 1734.

Krasko, V. A. and Doris, E. (2013). "state distributed pv policies: Can low cost (to government) policies have a market impact?", *Energy Policy* pp. 172 – 181.

Lee, M., Hong, T., Yoo, H., Koo, C., Kim, J., Jeong, K., Jeong, J. and Ji, C. (2017). "establishment of a base price for the solar renewable energy credit (srec) from the perspective of residents and state governments in the united states", *Renewable and Sustainable Energy Reviews* **7**: 1066 – 1080.

Maheu, J. M. and McCurdy, T. H. (2004). News Arrivals, Jump Dynamics and Volatility Components for Individual Stock Returns, *Journal of Finance* **59**: 755 – 793.

Mamkhezri, J., Thacher, J. A. and Chermak, J. M. (2020). Consumer Preferences for Solar Energy: A Choice Experiment Study, *The Energy Journal* **41**: 157 – 184.

Mason, C. F. (2001). Nonrenewable resources with switching costs, *Journal of Environmental Economics and Management* **42**: 65 – 81.

Mason, C. F. and Wilmot, N. A. (2016a). Jump Processes in Natural Gas Markets, *Energy Economics* **46**: S69 – S79.

Mason, C. F. and Wilmot, N. A. (2016b). "price discontinuities in the market for rins", *Journal of Economic Behavior & Organization* **132**: 79 – 97.

Mason, C. F. and Wilmot, N. A. (2020). Jumps in the convenience yield of crude oil, *Resource and Energy Economics* **60**: 101163.

Mason, C. F. and Wilmot, N. A. (2021). Energy Price Jumps, Fat Tails and Climate Policy, *Environmental Modeling & Assessment* . Forthcoming.

Merton, R. (1976). Option Pricing when the Underlying Stock Returns are Discontinuous, *Journal of Financial Economics* **3**: 125 – 144.

Muehlenbachs, L. (2015). A dynamic model of cleanup: Estimating sunk costs in oil and gas production, *International Economic Review* **56**: 155 – 185.

Nemet, G. F. (2019). *How Solar Energy Became Cheap: A Model for Low-Carbon Innovation*, 1 edn, Routledge.

O'Sullivan, M., Overland, I. and Sandalow, D. (2017). The Geopolitics of Renewable Energy, *Technical report*, Center on Global Energy Policy, Columbia University. Working Paper.

Paddock, J. L., Siegel, D. R. and Smith, J. L. (1988). Option Valuation of Claims on Real Assets: The Case of Offshore Petroleum Leases, *The Quarterly Journal of Economics* **103**(3): 479 – 508.

Panagiotidis, T. and Rutledge, E. J. (2007). Oil and gas markets in the U.K.: Evidence from a co-integrating approach, *Energy Economics* **29**: 329 – 347.

Pindyck, R. S. (2004). Volatility in Oil and Gas Markets, *The Journal of Energy and Development* **30**: 1 – 19.

Postali, F. and Picchetti, P. (2006). Geometric Brownian Motion and Structural Breaks in Oil Prices: A Quantitative Analysis, *Energy Economics* **28**: 506 – 522.

Sadorsky, P. (2006). Modeling and Forecasting Petroleum Futures Volatility, *Energy Economics* **26**: 467 – 488.

Sovacool, B. K. (2016). How Long Will It Take? Conceptualizing the Temporal Dynamics of Energy Transitions, *Energy Research & Social Science* **13**: 202 – 215.

Thompson, M., Davison, M. and Rasmussen, H. (2009). Natural gas storage valuation and optimization: A real options application, *Naval Research Logisitcs (NRL)* **56**: 226 – 238.

Torani, K., Rausser, G. and Zilberman, D. (2016). Innovation Subsidies Versus Consumer Subsidies: A Real Options Analysis of Solar Energy, *Energy Policy* **92**: 255 – 269.

U.S. Energy Information Administration (2021). Annual energy outlook 2021, with projections to 2050. https://www.eia.gov/outlooks/aeo/pdf/AEO_Narrative_2021.pdf.

van Benthem, A., Gillingham, K. and Sweeney, J. (2008). learning-by-doing and the optimal solar policy in california.

Verdolini, E., Anadón, L. D., Baker, E., Bosetti, V. and Aleluia Reis, L. (2018). Future Prospects for Energy Technologies: Insights from Expert Elicitations, *Review of Environmental Economics and Policy* **12**: 133 – 153.

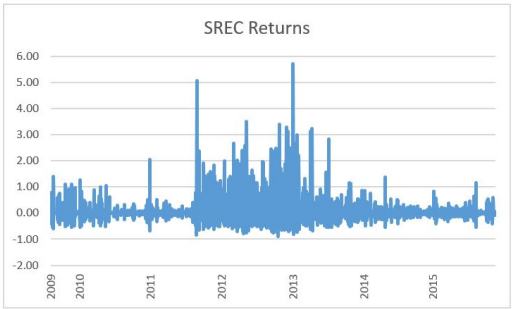
Wilmot, N. A. (2016). Discontinuities in the coal market, *Applied Economics Letters* **23**: 790–794.

Wilmot, N. A. and Mason, C. F. (2013). Jump Processes in the Market for Crude Oil, *The Energy Journal* **34**: 33 – 48.

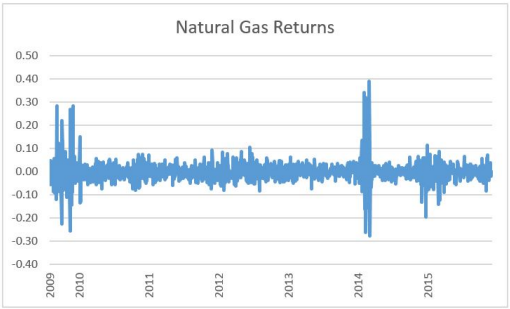
World Bank (2021). Private Participation in Infrastructure (PPI): 2021 Half Year (H1) Report. https://ppi.worldbank.org/content/dam/PPI/documents/PPI_2021_Half-Year-Report.pdf.

Xiaoming, L., Wan, K. and Ruiyu, L. (2012). Analysis on characteristics of coal price's volatility: Based on garch model, *Technology Economics* **4**: 12.

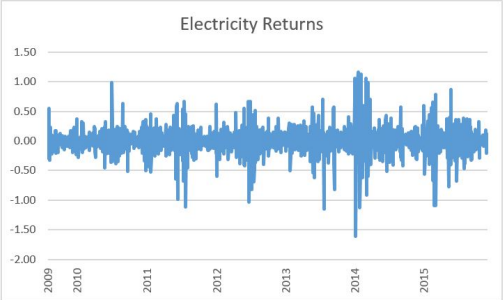
Figure 1: Time series of price returns



(a) SREC



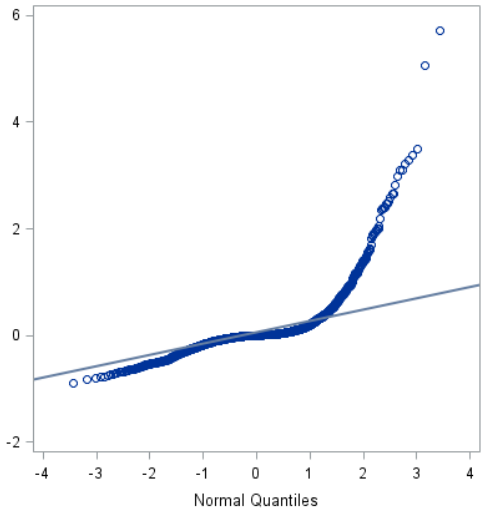
(b) Natural gas



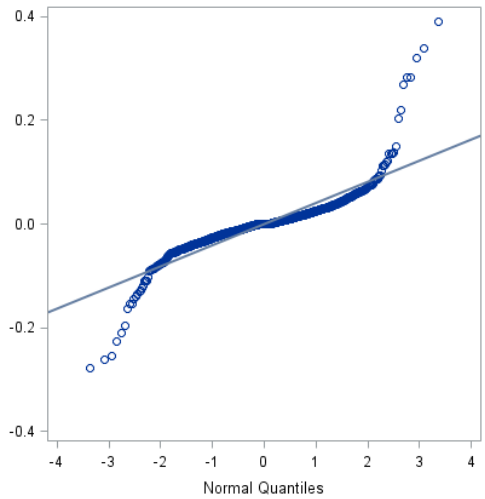
(c) Electricity

Source: Authors' calculations, based on data from
(a) https://njcleanenergy.com/renewable-energy/project-activity-reports/srec-pricing/srec-pricing-archive;
(b) https://www.eia.gov/dnav/ng/ng_pri_fut_s1_d.htm;
(c) https://www.eia.gov/electricity/wholesale/.

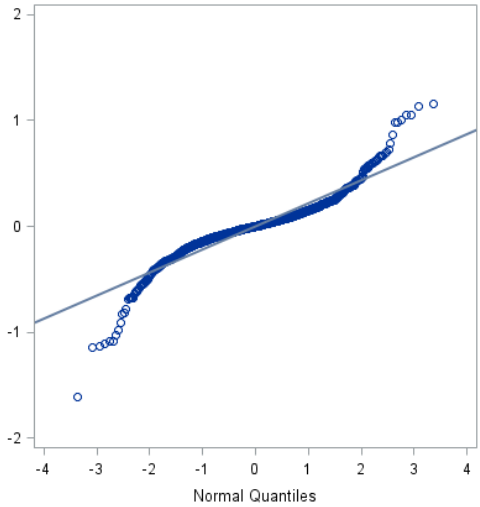
Figure 2: Quantile–quantile plots



(a) SREC



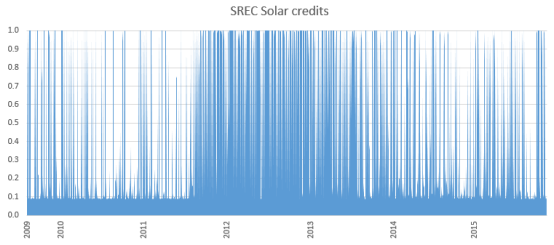
(b) Natural gas



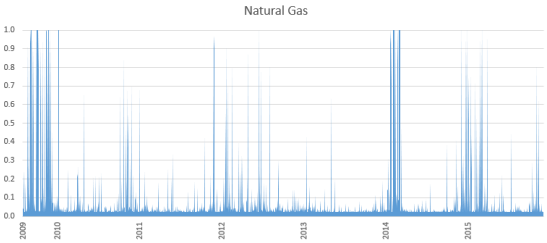
(c) Electricity

Source: Authors' calculations, based on data from
(a) https://njcleanenergy.com/renewable-energy/project-activity-reports/srec-pricing/srec-pricing-archive;
(b) https://www.eia.gov/dnav/ng/ng_pri_fut_s1_d.htm;
(c) https://www.eia.gov/electricity/wholesale/.

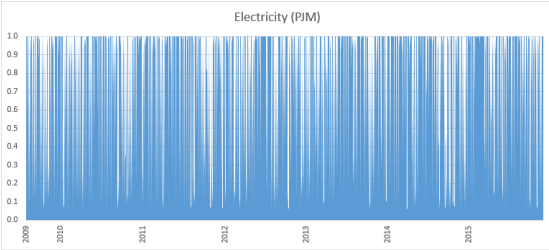
Figure 3: Fitted probabilities based on the Jump Diffusion model results



(a) SREC



(b) Natural gas



(c) Electricity

Source: Authors' calculations, based on results from Table 2.

Table 1: Summary Statistics: Solar Price Returns

Variable	Solar Returns	Natural Gas Returns	Electricity Returns
Sample Range			
Start	Aug 01, 2009	Aug 01, 2009	Aug 01, 2009
End	Nov 30, 2015	Nov 30, 2015	Nov 30, 2015
Summary Statistics			
Mean	0.0648	-0.00029	-0.0003
Median	-0.0014	0.00000	0.00000
Minimum	-0.8867	-0.27844	-1.6136
Maximum	5.7102	0.39007	1.1624
Variance	0.2137	0.00166	0.0471
Std. Dev.	0.4623	0.04071	0.2171
Coeff. of Variation	713.6	-14,172.5	-84,591.0
Skewness	4.1873	1.11170	-0.3023
Kurtosis	29.4956	20.22889	7.6107
n	2099	1599	1606
Test of Normality			
Kolmogoro Smirnov	0.2508	0.1133	0.1017
p-value	<0.01	<0.01	<0.01
Unit Root Test			
Modified Dickey Fuller	-4.806	-4.909	-4.446
1% critical-value†	-3.48	-3.48	-3.48
Lags	20	23	24

Note: Statistics are based on a sample of daily observations. Solar returns are based on SREC prices; natural gas returns based on Henry Hub spot prices; and electricity returns are based on PJM prices.
†: Critical values are based on Elliot et al. (1996).

Table 2: Estimation of the Parameters for Solar Price Returns

Variable	μ	σ	κ	α_1	β_1	λ	θ	δ
				Solar Credits				
PD	0.0648***	0.4622***	.	.	.	.	.	.
	(0.010)	(0.007)						
JD	-0.0069***	0.0536***	.	.	.	0.6673***	0.1074***	0.4540***
	(0.002)	(0.032)				(0.037)	(0.014)	(0.17)
GPD	0.0236***	.	0.0011***	0.1084***	0.8816***	.	.	.
	(0.004)		(0.000)	(0.009)	(0.009)			
GJD	0.0017	.	0.0007***	0.3899***	0.3216***	0.2065***	0.3818***	0.6744***
	(0.001)		(0.0001)	(0.058)	(0.050)	(0.019)	(0.056)	(0.045)
				Natural Gas				
PD	-0.02872	4.0695***	.	.	.	.	.	.
	(0.043)	(0.072)						
JD	-0.0529	2.3915***	.	.	.	0.0988***	0.2385***	10.3349***
	(0.046)	(0.076)				(0.019)	(0.416)	(9.63)
GPD	-0.0829	.	0.2716***	0.1404***	0.8431***	.	.	.
	(0.063)		(0.068)	(0.017)	(0.017)			
GJD	-0.0704	.	0.5013***	0.1160***	0.8010***	0.0195***	0.9577 ***	10.3349***
	(0.062)		(0.105)	(0.016)	(0.024)	(0.007)	(2.77)	(3.029)
				Electricity Prices				
PD	-0.0449	21.7033***	.	.	.	.	.	.
	(0.090)	(0.383)						
JD	0.2161	10.8724***	.	.	.	0.3169***	-0.7611	32.7599***
	(0.725)	(0.591)				(0.055)	(1.358)	(2.822)
GPD	0.3107	.	19.6518***	0.2463***	0.7350***	.	.	.
	(0.332)		(3.879)	(0.029)	(0.026)			
GJD	-0.8772	.	7.9596**	0.2219***	0.6945***	0.1654***	10.3549***	21.7218***
	(0.490)		(5.520)	(0.027)	(0.032)	(0.062)	(2.836)	(4.758)

† The standard errors are presented in the parentheses. ***, **, and * indicate statistical significance at the 1%, 5% and 10% levels, respectively.

Table 3: Likelihood Ratio Test Results

Variable	$LR_{PD,JD}$	$LR_{PD,GPD}$	$LR_{JD,GJD}$	$LR_{GPD,GJD}$
Solar Credits *(SERC)*	2539.02	1714.60	570.62	1395.04
	(0.000)	(0.000)	(0.000)	(0.000)
Natural Gas *(HH)*	812.45	999.34	254.05	67.16
	(0.000)	(0.000)	(0.000)	(0.000)
Electricity *PJM*	540.64	634.21	246.59	153.02
	(0.000)	(0.000)	(0.000)	(0.000)

Note: Based on a sample of 2009 observations. *p-values* are given in the parentheses.