Pricing of futures on CO_2 emission allowances: empirical analysis and forecasting performance

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Abstract

This paper analyzes the in- and out-of-sample performance of the one-factor model proposed in Moreno, Novales, and Platania (2019), that assumes that the prices of futures present mean reversion and seasonality. These features are modelled by Fourier series. We consider different specifications of this model and compare them versus the Schwartz (1997) and Lucía and Schwartz (2002) models. We also study the introduction of a second factor, the convenience yield. We use data of prices of futures on EUA (European Emission Allowances) from 2009 until 2018, traded in the Intercontinental Exchange. The main qualitative conclusions are: a) the in-sample behavior of the Moreno *et al.* (2019) model outperforms all the other one-factor models, b) there is not a clear winner model when analyzing the forecasting power, and c) the two-factor model fits better than any one-factor alternative.

Keywords: Fourier series, CO_2 Markets, Seasonality, Periodic fluctuations, Long-term swing.

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1 Introduction

Carbon dioxide is a greenhouse gas that causes the increase in global temperature. This gas has recently reached its highest level of atmospheric concentration in the last 800,000 years. Climate change is one of the biggest changes that occur globally. In an attempt to analyze the relevance of this asset class, this paper focuses on CO_2 emission allowances. In more detail, we will analyze the in- and out-of-sample behavior of several models on the prices of these allowances futures in order to find which continuous-time model fits better to the observed market prices of these futures.

Some authors have studied these CO_2 emission allowances and the prices of futures on these assets. For instance, Benz and Trück (2008), Paolella and Taschini (2008), and Seifert *et al.* (2008) performed an econometric analysis of the prices of emission allowances and study different models to determine the spot price dynamics. Chevallier (2009) analyzed the relationship between the profitability of carbon futures and changes in macroeconomic conditions. Bredin and Muckley (2011) studied how certain fundamental factors (such as economic growth, energy prices, and climatic conditions) can determine the prices of US futures. Conrad *et al.* (2011) modeled the adjustment process by which US prices respond to publications and announcements and found that the decissions of the European Commission on domestic allocation plans have a major impact on US prices.

In this paper, we focus on the model proposed in Moreno, Novales, and Platania (2019) (Moreno *et al.* (2019), from now on), which assumes that the prices of certain commodities can present mean-reversion and seasonality. The mean-reversion theory of commodity prices assumes that these prices revert to a certain long-term level: intuitively, if the price is greater than this level, the supply of the commodity tends to increase, and when the price is lower than the price at which they revert, the supply will decrease. This model was initially proposed for energy futures and its empirical behavior for agricultural futures has been analyzed in Del Campo-Bustos and Moreno (2018).

We also consider several versions of this model in order to study the possible relevance of an additional component of seasonality and the presence of long-term oscillations. It is expected that the introduction of Fourier series in the model, which reflects seasonality and convergence to a certain long-term level, can help to estimate the price of futures on CO_2 emissions. In addition, we also analyze futures prices by introducing a second factor, the convenience yield. For this, we consider the Bacaicoa *et al.* (2014) two-factor model. According to the financial theory the convenience yield decreases futures prices considering the effects of this yield on the underlying asset.

In this paper we will perform both in- and out-of-sample analysis. The in-sample analysis of one-factor models will be performed by applying non-linear least-squares regressions while the out-of-sample analysis of one-factor models and the in-sample analysis of the two-factor model will be based on the Kalman filter. All our analysis consider data from US futures (EU Allowances) from 2009 to 2018, which are traded in the Intercontinental Exchange (ICE).

The structure of this paper is as follows. In section 2 we describe the models that we will use in this paper. Section 3 presents the econometric methodology, section 4 describes the characteristics of the data under analysis and section 5 presents the empirical results. Finally, section 6 summarizes the main conclusions and suggests some possible future lines of research.

2 Models

This section introduces several one-factor models and one two-factor model. All these models are the starting point for the empirical analysis that will be presented subsequently.

2.1 One-factor models

2.1.1 Model 1 [Schwartz (1997)]

This author assumes that the spot price at time t of a certain commodity, S_t, follows a diffusion process given by

$$dS_t = \kappa(\mu - \ln(S_t))S_t dt + \sigma dW_t$$

where $\kappa,\ \mu,\ \sigma \in \mathbb{R}$ and W_t is a standard Wiener process. These (constant) parameters indicate, respectively, the speed of mean reversion, the long-term value of mean-reversion, and the diffusion coefficient of this process. It is worth to note that this process assumes no seasonality in this price.

Let $X_t = \ln(S_t)$ be the log-spot price. Applying the Itô's Lemma, this price follows an Ornstein-Uhlenbeck stochastic process that, under the equivalent martingale measure, can be written as:

$$dX_t = \kappa(\widetilde{\alpha} - X_t)dt + \sigma d\widetilde{W}_t \tag{1}$$

where $\widetilde{\alpha} = \mu - \frac{\sigma^2}{2\kappa} - \frac{\sigma\lambda}{\kappa}$, with a constant market price of risk, λ, and $d\widetilde{W}_t = W_t + \lambda t$ is a standard Wiener process under the risk-neutral measure.

Under this measure, the solution to equation (1) is given by:

$$X_s = e^{-\kappa(s-t)}X_t + (1 - e^{-\kappa(s-t)})\widetilde{\alpha} + \sigma \int_t^s e^{-\kappa(s-u)} d\widetilde{W}_u$$

Then, given the filtration $\mathscr{F}_t$, this variable follows a Gaussian distribution with mean and variance:

$$\begin{aligned}
\widetilde{E}[X_T|\mathscr{F}_t] &= e^{-\kappa(T-t)}\ln(S_t) + (1 - e^{-\kappa(T-t)})\widetilde{\alpha} \\
\widetilde{V}[X_T|\mathscr{F}_t] &= \widetilde{V}\left(\sigma \int_t^T e^{-\kappa(T-t)} d\widetilde{W}_t\right) = \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa(T-t)})
\end{aligned}$$

Therefore, applying the properties of the lognormal distribution, the forward price of the commodity

at time T is the expected commodity price under the martingale measure:

$$\begin{aligned} F(S_t,t,T) &= \widetilde{E}[S_T|\mathscr{F}_t] = \exp\left\{\widetilde{E}[X_T|\mathscr{F}_t] + \frac{1}{2}\widetilde{V}[X_T|\mathscr{F}_t]\right\} \\ &= \exp\left\{e^{-\kappa(T-t)}\ln(S_t) + (1-e^{-\kappa(T-t)})\widetilde{\alpha} + \frac{\sigma^2}{4\kappa}(1-e^{-2\kappa(T-t)})\right\} \end{aligned}$$

Taking logarithms, we get the following expression:

$$\ln(F(S_t,t,T)) = e^{-\kappa(T-t)}\ln(S_t) + (1-e^{-\kappa(T-t)})\widetilde{\alpha} + \frac{\sigma^2}{4\kappa}(1-e^{-2\kappa(T-t)})$$

2.1.2 Model 2 [Lucía and Schwartz (2002)]

These authors analyze the possible existence of regular patterns in the behavior of electricity prices by analyzing spot and futures prices in the Nord Pool market. This regular (seasonal) pattern in the futures curve will be modelled with a sinusoidal function. Then, the process for spot prices includes two components: a) a deterministic component that explains the regularities in the price evolution and b) a stochastic continuous-time process.[1]

In more detail, the log-spot price process can be written as:

$$\ln(S_t) = f_t + Y_t$$

where f_t is a time-varying deterministic function and Y_t follows a mean-reverting process, whose dynamics is given by:

$$dY_t = -\kappa Y_t dt + \sigma dW_t$$

Assuming a constant risk market price, λ, the risk-neutral version of this process is:

$$dY_t = \kappa(\alpha^* - Y_t)dt + \sigma d\widetilde{W}_t,\ \alpha^* = -\frac{\sigma\lambda}{\kappa}$$

The function f_t aims to capture the behavioral components of electricity prices derived from natural regularities along time. Therefore, the seasonal behavior will be described by a process that fluctuates randomly around a constant long-term mean value. This function will include a constant plus two terms, that aim to capture, respectively, the variation in the price level distinguishing between working and non-working days and the seasonal yearly evolution of prices. In more detail, we have

$$f_t = \alpha + \beta D_t + \gamma\cos\left((t+\tau)\cdot\frac{2\pi}{365}\right)$$

where $\alpha,\ \beta,\ \gamma,\ \tau \in \mathbb{R}$ and the dummy D_t equates one in week-ends or holidays and is zero otherwise. The cosine function reflects the seasonal pattern in the price evolution along the year.

[1] For simplicity, constant interest rates are considered.

Under the risk-neutral probability, the log-spot price $X_t = \ln(S_t)$ is given by:

$$X_s = f_s + e^{-\kappa(s-t)}Y_t + (1 - e^{-\kappa(s-t)})\alpha^* + \sigma \int_t^s e^{-\kappa(s-u)} d\widetilde{W}_u, \; s > t$$

Then, the log-spot is normally distributed and, then, the spot price follows a lognormal distribution. Finally, the price at time t of a futures on this commodity that matures at T is given by:

$$\begin{aligned} F(S_t, t, T) &= \widetilde{E}[S_T|\mathscr{F}_t] = \exp\left\{\widetilde{E}[X_T|\mathscr{F}_t] + \frac{1}{2}\widetilde{V}[X_T|\mathscr{F}_t]\right\} \\ &= \exp\left\{f_T + e^{-\kappa(T-t)}(\ln(S_t) - f_t) + \left(1 - e^{-\kappa(T-t)}\right)\alpha^* + \frac{\sigma^2}{4\kappa}\left(1 - e^{-2\kappa(T-t)}\right)\right\} \end{aligned}$$

2.1.3 Moreno, Novales, and Platania (2019) model

These authors present a model for commodities prices and obtain closed-form expressons for prices of different derivatives, including futures. They incorporate seasonal and cyclical fluctuations as well as long-term fluctuations in futures prices. One of their main assumptions is that futures prices converge to a long-term value that experiences periodic and smooth fluctuations throughout certain time periods. For cyclical and seasonal fluctuations they introduce deterministic terms in the stochastic process of the log spot price.

In a similar way to Lucía and Schwartz (2002), these authors assume that the process for the spot price can be split into two components

$$\ln(S_t) = f(t) + Y_t$$

The deterministic component $f(t)$ that represents the seasonal behavior of the commodity price is modeled with a Fourier series:

$$f(t) = \sum_{n=0}^{\infty} Re[A_n e^{inw_f t}]$$

The component Y_t follows a mean-reverting process, which reverts to $z(t)$, that depends on time and captures long-term variations:

$$\begin{aligned} dY_t &= \kappa(z(t) - Y_t)dt + \sigma dW_t \\ z(t) &= \sum_{n=0}^{\infty} Re\left[B_n e^{inw_z t}\right] \end{aligned}$$

where $\kappa,\ \sigma,\ w_f,\ w_z \in \mathbb{R}^+$ and W_t is a standard Wiener process. A_n and B_n are complex numbers: $A_n = A_{x,n} + iA_{y,n}$ and $B_n = B_{z,n} + iB_{y,n}$. If $B_n = 0$ and $A_n = 0$, this model nests that in Schwartz (1997). This model also nests the Lucía and Schwartz (2002) model if $B_n = 0$ and $f(t)$ matches the function proposed by those authors.

The futures price of a commodity is obtained using the properties of the lognormal distribution under the risk-neutral measure:

$$\begin{aligned} F(S_t,t,T) &= \tilde{E}[S_t|F_t] = \exp\left\{\tilde{E}[\ln(S_t)|F_t] + \frac{1}{2}\tilde{V}[\ln(S_t)|F_t]\right\} \\ &= \exp\left\{f(T) + e^{-\kappa(T-t)}(\ln(S_t) - f(t)) + (1-e^{-\kappa(T-t)})\alpha + \frac{\sigma^2}{4\kappa}(1-e^{-2\kappa(T-t)})\right. \\ &\quad \left. + \sum_{n=1}^{\infty} Re\left[\frac{\kappa B_n}{\kappa + inw_z}(e^{inw_zT} - e^{-\kappa(T-t)+inw_zt})\right]\right\} \end{aligned}$$

that, taking logarithms, can be written as

$$\begin{aligned} \ln(F(S_t,t,T)) &= f(T) + e^{-\kappa(T-t)}(\ln(S_t) - f(t)) + (1-e^{-\kappa(T-t)})\alpha + \frac{\sigma^2}{4\kappa}(1-e^{-2\kappa(T-t)}) \\ &\quad + \sum_{n=1}^{\infty} Re\left[\frac{\kappa B_n}{\kappa + inw_z}(e^{inw_zT} - e^{-\kappa(T-t)+inw_zt})\right] \end{aligned}$$

Then, the logarithm of the futures price can be split into four components:

- Spot effect, indicating a correction of the logarithm of the spot price:

$$e^{-\kappa(T-t)}\ln(S_t)$$

- Seasonal and cyclical effect, that quantifies the effect of the periodic, seasonal, and cyclical component (short- and medium-term effects):

$$f(T) - e^{-\kappa(T-t)}f(t)$$

- Volatility effect:

$$\frac{\sigma^2}{4\kappa}\left(1-e^{-2\kappa(T-t)}\right)$$

- Long-term (swing) effect, that is, the effect of the mean-reversion level:

$$(1-e^{-\kappa(T-t)})\alpha + \sum_{n=1}^{\infty} Re\left[\frac{\kappa B_n}{\kappa + inw_z}(e^{inw_zT} - e^{-\kappa(T-t)+inw_zt})\right]$$

Therefore, the logarithm of the futures price can be expressed as the sum of two terms:

$$\ln(F_t(S_t,t,T)) = M(S_t,t,T;\theta) + N(t,T;\theta)$$

where

$$M(S_t,t,T;\theta) = e^{-\kappa(T-t)}\ln(S_t) + (1-e^{-\kappa(T-t)})\alpha + \frac{\sigma^2}{4\kappa}(1-e^{-2\kappa(T-t)})$$

depends on the spot price and time to expiration. On the other hand, we have

$$N(t,T;\theta) = \sum_{n=0}^{\infty} Re\left[A_n(e^{inw_f T} - e^{-\kappa(T-t)+inw_f t})\right] + \sum_{n=1}^{\infty} Re\left[\frac{\kappa B_n}{\kappa + inw_z}(e^{inw_z T} - e^{-\kappa(T-t)+inw_z t})\right]$$

that represents the seasonality component, which is the novel aspect of this model with respect to previous ones. The two terms of this equation describe, respectively, the seasonal fluctuations around the long-term mean price and the time evolution of the level of mean-reversion.

We will analyze seven particular cases of this model:

- Model 3 incorporates a single term of the Fourier series to represent the level of mean-reversion, so that the second component of the logarithm of the futures price is given by:

$$N(t,T;\theta) = Re\left[\frac{\kappa B}{\kappa + iw_z}(e^{iw_z T} - e^{-\kappa(T-t)+iw_z t})\right]$$

- Model 4 adds a Fourier series representation with a single frequency for $f(t)$, with the goal of capturing the cyclical component and seasonal, so the second component will be:

$$N(t,T;\theta) = Re[A(e^{iw_{f,1}T} - e^{-\kappa(T-t)+iw_{f,1}t})] + Re\left[\frac{\kappa B}{\kappa + iw_z}(e^{iw_z T} - e^{-\kappa(T-t)+iw_z t})\right]$$

- Model 5 considers two different frequencies in the series $f(t)$, aiming to capture fluctuations in the business cycle and short-term seasonality. So, we have:

$$N(t,T;\theta) = \sum_{l=1,2} Re[A_l(e^{iw_{f,l}T} - e^{-\kappa(T-t)+iw_{f,l}t})] + Re\left[\frac{\kappa B}{\kappa + iw_z}(e^{iw_z T} - e^{-\kappa(T-t)+iw_z t})\right]$$

- Model 6 considers three frequencies in $f(t)$ trying to improve the fit to observed data. In this case, we have:

$$N(t,T;\theta) = \sum_{l=1,2,3} Re[A_l(e^{iw_{f,l}T} - e^{-\kappa(T-t)+iw_{f,l}t})] + Re\left[\frac{\kappa B}{\kappa + iw_z}(e^{iw_z T} - e^{-\kappa(T-t)+iw_z t})\right]$$

- Model 7 considers a single frequency for $f(t)$ and does not take into account the representation of the level of mean-reversion. Therefore, we have:

$$N(t,T;\theta) = Re[A(e^{iw_{f,1}T} - e^{-\kappa(T-t)+iw_{f,1}t})]$$

- Model 8 incorporates two frequencies in the $f(t)$ series that capture the fluctuations of the business cycle and short-term seasonality:

$$N(t,T;\theta) = \sum_{l=1,2} Re[A_l(e^{iw_{f,l}T} - e^{-\kappa(T-t)+iw_{f,l}t})]$$

- Model 9 adds a third frequency for $f(t)$ trying to capture better the seasonality, so we have:

$$N(t,T;\theta) = \sum_{l=1,2,3} Re[A_l(e^{iw_{f,l}T} - e^{-\kappa(T-t)+iw_{f,l}t})]$$

Moreno *et al.* (2019) estimated empirically models 1 to 5 using prices of three energy futures and found that their models (3 to 5) outperformed models 1 and 2.

2.2 Bacaicoa, Moreno, and Platania (2014) model

These authors extend the Moreno *et al.* (2019) model to two factors, these factors being the spot price and the convenience yield, which follow the processes

$$\begin{aligned} dS_t &= (\mu - \delta_t)S_t dt + \sigma_1 S_t dW_t^1 \\ d\delta_t &= \kappa(f(t) - \delta_t)dt + \sigma_2 dW_t^2 \end{aligned}$$

where $W_t^i,\ i = 1, 2$ are correlated standard Brownian motions with $dW_t^1 dW_t^2 = \rho dt$.

The convenience yield converges in the long-term to the function $f(t)$, characterized by a Fourier series

$$f(t) = \sum_{n=0}^{\infty} Re[A_n e^{inwt}]$$

Solving these processes, the convenience yield is given by the following process

$$\begin{aligned} \delta_T &= e^{-\kappa(T-t)}\delta_t + \alpha(1 - e^{-\kappa(T-t)}) + \kappa \sum_1^{\infty} Re\left[\frac{A_n}{inw+\kappa}(e^{inwT} - e^{inwt-\kappa(T-t)})\right] \\ &\quad + \sigma_2 \int_t^T (1 - e^{-\kappa(T-u)})d\tilde{W}_u^2 \end{aligned}$$

Therefore, the futures price is given by

$$\begin{aligned} F(S_t,t,T) &= S_t \exp\left\{\left(r - \alpha + \frac{\sigma_2^2}{2\kappa^2} - \frac{1}{\kappa}\sigma_1\sigma_2\rho\right)(T-t)\right. \\ &\quad + \frac{1-e^{-\kappa(T-t)}}{\kappa}\left(\alpha - \delta_t + \frac{1}{\kappa}\sigma_1\sigma_2\rho - \frac{\sigma_2^2}{\kappa^2}\right) + \frac{1-e^{-2\kappa(T-t)}}{\kappa^3}\frac{\sigma_2^2}{4\kappa^3} \\ &\quad \left. + \sum_1^{\infty} Re\left[\frac{A_n}{inw+\kappa}\left(e^{inwT} - e^{inwt-\kappa(T-t)}\right)\right] - \sum_1^{\infty} Re\left[\frac{A_n}{inw}\left(e^{inwT} - e^{inwt}\right)\right]\right\} \end{aligned}$$

or, equivalently,

$$\begin{aligned} \ln(F(S_t,t,T)) &= \ln(S_t) + \left(r - \alpha + \frac{\sigma_2^2}{2\kappa^2} - \frac{1}{\kappa}\sigma_1\sigma_2\rho\right)(T-t) \\ &\quad + \frac{1-e^{-\kappa(T-t)}}{\kappa}\left(\alpha - \delta_t + \frac{1}{\kappa}\sigma_1\sigma_2\rho - \frac{\sigma_2^2}{\kappa^2}\right) + \frac{1-e^{-2\kappa(T-t)}}{\kappa^3}\frac{\sigma_2^2}{4\kappa^3} \\ &\quad + \sum_1^{\infty} Re\left[\frac{A_n}{inw+\kappa}\left(e^{inwT} - e^{inwt-\kappa(T-t)}\right)\right] - \sum_1^{\infty} Re\left[\frac{A_n}{inw}\left(e^{inwT} - e^{inwt}\right)\right] \end{aligned}$$

3 Econometric method

We describe now the optimization problem and the econometric methodology that will be implemented to estimate the parameters of the different (one- or two factor) models that were presented in the previous section.

3.1 Optimization problem

The optimization problem to be expressed as

$$\min(SR(\tilde{\theta};\tilde{\theta}_m)) = \sum_{t=1}^{T}\sum_{j=1}^{\kappa}(P_{jt} - \beta^{'}\eta_{j,t})^{'}W(P_{jt} - \beta^{'}\eta_{j,t})$$

where $W = I_k$ and

$$\beta^{'}\eta_{j,t} = \sum_{i=1}^{8}\beta_i\eta_{ij,t}$$

The price of each commodity presents a common structure for each futures but, in addition, there may be specific seasonal and cyclical components for each maturity of the corresponding futures (models 4 to 9).

We have:

$$\begin{aligned} P_t &= \ln(F(S_t,t,T)) - e^{\kappa(T-t)}\ln(S_t) = \sum_{i=1}^{10}\beta_i\eta_{ij,t} + u_t \\ \eta_{1t} &= 1 - e^{-\kappa(T-t)} \\ \eta_{2t} &= \frac{1}{4\kappa}(1 - e^{-2\kappa(T-t)}) \end{aligned}$$

Therefore, P_t and η_{it} are non-linear functions of the structural parameters. It is important to indicate that $\eta_{it},\ i = 3,\cdots,10$ present a different specification for each model.

In order to separate the long-term common component and the specific components of the different maturities, we need to impose a common level of mean-reversion for all the maturities but allowing seasonal and cyclical components to differ across maturities.

For each model, we have the following parameters:

- **Model 1**: Schwartz (1997):

$$\beta_1 = \tilde{\alpha} = \mu - \frac{\sigma^2}{2\kappa} - \frac{\sigma\lambda}{\kappa},\ \beta_2 = \sigma^2,\ \beta_i = 0,\ i = 3,\cdots,8$$

- **Model 2**: Lucía and Schwartz (2002). This model incorporates a seasonality component:

$$\begin{aligned} \beta_1 &= \tilde{\alpha} = \mu - \frac{\sigma\lambda}{\kappa},\ \beta_2 = \sigma^2, \quad \beta_3 = \gamma,\ \beta_i = 0,\ i = 4,\cdots,8 \\ \eta_{3t} &= \cos\left((T+\varphi)\frac{2\pi}{260}\right) - e^{\kappa(T-t)}\cos\left((t+\varphi)\frac{2\pi}{260}\right) \end{aligned}$$

We now study the three specifications proposed by Moreno *et al.* (2019):

- **Model 3** incorporates long-term oscillations, where

$$\beta_1 = \alpha,\ \beta_2 = \sigma^2,\ \beta_i = 0,\ i = 5, \cdots, 10$$

  Being the Fourier components

$$\beta_3\eta_{3t} + \beta_4\eta_{4t} = Re\left[(B_x + iB_y)\frac{\kappa}{\kappa + iw_z}(e^{iw_zT} - e^{-\kappa(T-t)+iw_z\kappa t})\right]$$

- **Model 4** includes long-term oscillations and a seasonality component. So, we have:

$$\beta_1 = \alpha,\ \beta_2 = \sigma^2,\ \beta_7 = \beta_8 = \beta_9 = \beta_{10} = 0$$

  The Fourier components are:

$$\begin{aligned}\beta_3\eta_{3t} + \beta_4\eta_{4t} &= Re\left[(B_x + iB_y)\frac{\kappa}{\kappa + iw_z}(e^{iw_zT} - e^{-\kappa(T-t)+iw_z\kappa t})\right]\\ \beta_5\eta_{5t} + \beta_6\eta_{6t} &= Re\left[(A_{x,1} + iB_{y,1})e^{iw_{f,1}t}\right]\end{aligned}$$

- **Model 5** incorporates two terms in the Fourier series, related to the cyclical component, and a term in the long-term mean.

  In this case, $\beta_1 = \alpha,\ \beta_2 = \sigma^2,\ \beta_9 = \beta_10 = 0$, being the Fourier components

$$\begin{aligned}\beta_3\eta_{3t} + \beta_4\eta_{4t} &= Re\left[(B_x + iB_y)\frac{\kappa}{\kappa + iw_z}(e^{iw_zT} - e^{-\kappa(T-t)+iw_z\kappa t})\right]\\ \sum_{i=5}^{8}\beta_i\eta_{it} &= \sum_{l=1,2} Re\left[(A_{x,l} + iA_{y,l})e^{iw_{f,l}t}\right]\end{aligned}$$

  Models 4 and 5 incorporate more than one Fourier series. The lowest frequency is related to long-term oscillations in the level of mean-reversion and the remaining frequencies define the cyclical or seasonal components of futures prices.

  We present now models 6 to 9, which provide an extension of models 3 to 5:

- **Model 6** adds an additional seasonal component to model 5.

  The Fourier components are now given by

$$\begin{aligned}\beta_3\eta_{3t} + \beta_4\eta_{4t} &= Re\left[(B_x + iB_y)\frac{\kappa}{\kappa + iw_z}(e^{iw_zT} - e^{-\kappa(T-t)+iw_z\kappa t})\right]\\ \sum_{i=5}^{10}\beta_i\eta_{it} &= \sum_{l=1,2,3} Re\left[(A_{x,l} + iA_{y,l})e^{iw_{f,l}t}\right]\end{aligned}$$

- **Model 7** is equal to model 4 eliminating the long-term oscillations.

  Then, it includes just one seasonality component. So, we have:

$$\beta_1 = \alpha,\ \beta_2 = \sigma^2,\ \beta_i = 0,\ i = 7, \cdots, 10$$

  Being the Fourier components

$$\beta_5 \eta_{5t} + \beta_6 \eta_{6t} = Re\left[(A_{x,1} + iB_{y,1})e^{iw_{f,1}t}\right]$$

- **Model 8** incorporates two terms in the Fourier series, related to the cyclical component.

  In this case,

$$\beta_1 = \alpha,\ \beta_2 = \sigma^2,\ \beta_3 = \beta_4 = \beta_9 = \beta_{10} = 0$$

  Being the Fourier components

$$\sum_{i=5}^{8} \beta_i \eta_{it} = \sum_{l=1,2} Re\left[(A_{x,l} + iA_{y,l})e^{iw_{f,l}t}\right]$$

- **Model 9** adds one more seasonal component to model 8, so the Fourier components are now

$$\sum_{i=5}^{10} \beta_i \eta_{it} = \sum_{l=1,2,3} Re\left[(A_{x,l} + iA_{y,l})e^{iw_{f,l}t}\right]$$

We will apply the Kalman filter to perform the forecasting analysis of one-factor models and analyze the in-sample behavior of the two-factor model. Then, this technique will be briefly described before applying it to the different models.

Following Harvey (1989), the Kalman filter is a recursive algorithm, which allows to evaluate numerically a likelihood function to estimate certain parameters, generating estimates of the unobservable variables included in the model. The limitation of this method is that it assumes that no observable variable can affect unobservable states or variables.

To apply this algorithm we need to write the model in the form of state space, such that:

$$\begin{aligned} \xi_{t+1} &= F\xi_t + \upsilon_{t+1} \\ y_t &= A'x_t + H'\xi_t + \omega_t \end{aligned}$$

These equations are named state and observation equations, respectively. The vector ξ_t includes the unobservable variable while y_t and x_t include the observable variables. In more detail, x_t contains exogenous variables.

The goal is to predict ξ_{t+1} e y_{t+1} with the information available at time t, the prediction being $\hat{\xi}_{t+1|t} = E(\xi_{t+1}|Y_t)$, where Y_t denotes the set of available information.

3.2 Forecasting Analysis

All our models assume that there is a relationship between spot and futures prices, although the spot market tends to present more abrupt changes than the futures market. This is because, in the spot market, the closing prices are liquidation prices while, in the futures market, the average prices along the last minutes of the session are considered. For this reason, we will consider that spot prices can incorporate a noise component.

To perform the forecasting analysis of the one-factor models, the Kalman filter technique will be used, where the representation in the state space is as follows:

$$\begin{aligned} \ln(S_{t+1}) &= G + F\ln(S_t) + \upsilon_{t+1} \\ \ln(F(S_t,t,T)) &= A'x_t + H'\ln(S_t) + \omega_t \end{aligned}$$

where $E(\upsilon_t\upsilon_t') = Q$ and $E(\omega_t\omega_t') = R$, being Q and R constants.

We take the log-spot price at time $t+1$ as an unobservable variable in the state space representation. The process to be performed is as follows: at time t, the filtered spot price, S_t, and the estimates of $\hat{F}$ and $\hat{G}$ are used to predict S_{t+1}, which will be used to estimate the observation equation one period later, $\hat{F}_{t+1}$.

The observation equations, specific for each model, are the following:

- **Model 2** : Lucía and Schwartz (2002):

$$\ln(F(S_t,t,T)) = \begin{pmatrix} \alpha & \frac{\sigma^2}{4\kappa} & \gamma & \gamma \end{pmatrix} \begin{pmatrix} 1-e^{-\kappa(T_i-t)} \\ 1-e^{-2\kappa(T_i-t)} \\ \cos((T_i+\varphi)2\pi/269) \\ -e^{-\kappa(T_i-t)}\cos((t+\varphi)2\pi/260) \end{pmatrix} + e^{-\kappa(T_i-t)}\ln(S_t) + \omega_{ti}$$

- **Remaining models:**

  Remaining models can be summarized in a single matrix since, for each model, we get a particular case of a more general matrix:

$$\ln(F(S_t,t,T)) = A\,x_i' + e^{-\kappa(T_i-t)}\ln(S_t) + \omega_{ti}$$

  where

$$A = \begin{pmatrix} \alpha & \dfrac{\sigma^2}{4\kappa} & B_x & B_y & A_x & A_y & A_{x,2} & A_{y,2} & A_{x,3} & A_{y,3} \end{pmatrix}$$

and

$$x_i' = \begin{pmatrix} 1 - e^{-\kappa(T_i - t)} \\ \\ 1 - e^{-2\kappa(T_i - t)} \\ \\ \kappa\cos(\omega_z T_i) + \omega_z \sin(\omega_z T_i) - e^{-\kappa(T_i - t)}[\kappa\cos(\omega_z t) + \omega_z \sin(\omega_z t)] \\ \\ -\kappa\sin(\omega_z T_i) + \omega_z \cos(\omega_z T_i) - e^{-\kappa(T_i - t)}[\omega_z \cos(\omega_z t) - \kappa\sin(\omega_z t)] \\ \\ \cos(\omega_f T_i) - e^{-\kappa(T_i - t)}\cos(\omega_f t) \\ \\ e^{-\kappa(T_i - t)}\sin(\omega_f t) - \sin(\omega_f T_i) \\ \\ \cos(\omega_{f2} T_i) - e^{-\kappa(T_i - t)}\cos(\omega_{f2} t) \\ \\ e^{-\kappa(T_i - t)}\sin(\omega_{f2} t) - \sin(\omega_{f2} T_i) \\ \\ \cos(\omega_{f3} T_i) - e^{-\kappa(T_i - t)}\cos(\omega_{f3} t) \\ \\ e^{-\kappa(T_i - t)}\sin(\omega_{f3} t) - \sin(\omega_{f3} T_i) \end{pmatrix}$$

So we have:

- Model 1 includes the first two elements of both A and x_i.
- Model 3 considers the first four elements of both A and x_i.
- Model 4 consists of the first six elements of both A and x_i.
- Model 5 corresponds to the first eight elements of both A and x_i.
- Model 6 includes the first ten elements of both A and x_i.
- Model 7 does not take into account the third and fourth elements of both A and x_i because they correspond to long-term swings. Thus, model 7 has only one seasonal component, that is, it considers the first, second, fifth, and sixth elements of both A and x_i.
- Model 8 takes into account the elements of the model 7 and also the seventh and eighth elements of both A and x_i.
- Model 9 enlarges the model 8 by adding the ninth and tenth elements of both A and x_i.

We will perform the forecasting analysis for two different years, 2016 and 2017. We start predicting the first quarter of 2017. For this, we use daily data from 2016 to calibrate each model, except the frequency parameters for which we use the complete sample until the end of 2016. Once the model is calibrated, we obtain the one-day ahead forecasts for futures prices during the first quarter of 2017. Later, we recalibrate the parameters of each model using data from April, 2016 to March, 2017. For the frequency parameters we use the entire sample until March, 2016. And we forecast the second quarter of 2017. We continue this process until we get one-day ahead predictions for futures prices along the four quarters of 2017.

We perform the same procedure but using 2015 and 2016 data instead of just data from 2016. Thus, we forecast the first quarter of 2017 by using data from both years to calibrate the model and data from the whole sample until the end of 2016 to estimate the frequency parameters. The second quarter of 2017 is forecasted by using data from April, 2015 to March, 2017. And so on until we have predictions for the four quarters of 2017.

We apply the same method to forecast the year 2016, using alternatively data from 2015 and data from 2014 and 2015. We will compare our forecasted values versus observed data and we obtain the forecasting errors, that will be shown in the corresponding tables and discussed accordingly.

3.3 Two-factor model

The difficulty of this model with respect to one-factor models is due to the existence of unobservable variables, in this case, the convenience yield. As dicussed previously, for models with unobservable variables, the Kalman filter will be applied to estimate their parameters.

For the two-factor model under analysis (Bacaicoa *et al.*, 2014), the state space representation is the following:

- The state equation is

$$d\delta_t = \kappa(f(t) - \delta_t)dt + \sigma_2 dW_t^2$$

  The discrete version of this equation is

$$\delta_{t+\Delta t} - \delta_t = \kappa f(t)\Delta t - \kappa\delta_t\Delta t + \xi_t^2$$

  or, equivalently,

$$\delta_{t+\Delta t} = \kappa f(t)\Delta t + (1 - \kappa\Delta t)\delta_t + \xi_t^2$$

  where ξ_t^2 follows a distribution $N(0, \sigma_1^2\Delta t)$.

  Therefore, in matrix form, the state equation is

$$\underbrace{\delta_t}_{\alpha_t} = \underbrace{\kappa f(t)\Delta t}_{c_t} + \underbrace{(1 - \kappa\Delta t)}_{Q_t}\underbrace{\delta_{t-\Delta t}}_{\alpha_{t-\Delta t}} + \xi_t^2$$

- For this model, the observation equation is given by:

$$\begin{aligned}\ln(F(S_t,\tau)) &= -\frac{1-e^{-\kappa(\tau)}}{\kappa}\delta_t + X_t + \left(r-\alpha+\frac{\sigma_2^2}{2\kappa^2}-\frac{1}{\kappa}\sigma_1\sigma_2\rho\right)\tau \\ &+\frac{1-e^{-2\kappa\tau}}{\kappa^3}\frac{\sigma_2^2}{4\kappa^3}+\frac{1-e^{-\kappa\tau}}{\kappa}\left(\alpha+\frac{1}{\kappa}\sigma_1\sigma_2\rho-\frac{\sigma_2^2}{\kappa^2}\right) \\ &+\sum_1^\infty Re\left[\frac{A_n}{inw+\kappa}(e^{inw(t+\tau)}-e^{inwt-\kappa\tau})\right] \\ &-\sum_1^\infty Re\left[\frac{A_n}{inw}(e^{inw(t+\tau)}-e^{inwt})\right]+\varepsilon_t\end{aligned}$$

that can be rewritten as

$$Y_t = \underbrace{-\frac{1-e^{-\kappa(\tau)}}{\kappa}}_{H'}\underbrace{\alpha_t}_{\xi_t}+\underbrace{d_t}_{A'x_t}+\underbrace{\varepsilon_t}_{\omega_t}$$

where $Y_t = \ln(F(S_t,\tau))$, $\alpha_t = \delta_t$, and ε_t follow a distribution $N(0,h)$.

Once we have written the model in the state space form, we will apply the Kalman filter to estimate the parameters. As explained earlier, this filter is a recursive algorithm, which starts from initial values of $\hat{\alpha_{ti}} = c_t + Q_t\hat{\alpha_{ti-1}}$ and the variance-covariance matrix of the estimation error $\hat{P_{ti}} = Q_tP_{ti-1}Q_t' + V_t$.

4 Data

As mentioned above, we will use data for futures on EUA (European Emission Allowances). These futures have monthly maturities and, in each contract, 1,000 permits of CO_2 emissions are traded. Each permit allows one ton of carbon dioxide to be emitted and it is traded in euros and cents per metric ton. The minimum tick is 0.01€ per ton, or equivalently, 10€ for each futures. The futures expires at the last Monday of the corresponding month.[2] The closing price is computed as the average of the quoted prices during the daily closing period (16:50:00 - 16:59:59 hours, local time in United Kingdom), or with quoted liquidation prices if the liquidity is low. Contracts are physically settled and delivered by transferring the EU allowances from the seller account to the buyer accounts in the Union Registry. All transfers go through the account of the clearing member and ICE Clear Europe. The delivery is made three days after the last trading day.

Data series have been obtained from Datastream and closest-to-maturity futures will be used. For all the maturities, the sample period ends in March 9, 2018. The beginning of the sample period depends on the considered maturity. Specifically, we have:

[2]However, if the last Monday is a non-working day or there is a non-working day in the 4 days after the last Monday, the last trading day will be the penultimate Monday of the delivery month.

- Spot price and first and second closest-to-maturity futures: October 27, 2009.
- Third and fourth closest-to-maturity futures: July 30, 2010.
- Fifth closest-to-maturity futures: December 29, 2010.
- Sixth and seventh closest-to-maturity futures: April 3, 2012.

We perform a descriptive analysis of the prices of futures on CO_2 emission allowances. Table 1 provides several statistics of the prices of the closest-to-maturity futures and of the spot price. We see that the mean price of these futures ranges between 6.91 and 8.55, an interval that includes the mean spot price. We also see that the volatility has increased in the five closest maturities and is smaller in the sixth and seventh closest-to-maturity futures. This fact can be explained by the seasonality, that implies that the periods with higher amount of information relative to the level of production also present the highest volatility in all the maturities. Then, in the futures that mature after these periods, we can observe intervals of volatility increasing with maturity.

[INSERT TABLE 1 AROUND HERE]

In all the cases, we see a positive skewness, indicating that extreme high values have higher probability than low ones. This can be also checked in the maximum and minimum values as the maximum values are more extreme than minimum ones. The kurtosis ranges between 2.27 and 5.80 and increases with maturity except in maturitiess 6 and 7.

Figure 1 includes the time series for the spot price and futures prices and shows a similar evolution in all the maturities. From mid 2015 on, the time evolution in all the series is so similar that they are almost indistinghuisible. These futures reach maximum and values around, respectively, May, 2011 and April, 2013. In addition, the market seems to be in contango in most of the sample period.

[INSERT FIGURE 1 AROUND HERE]

Finally, we have studied the stationarity of these prices (in levels and in first differences) by computing the augmented Dickey-Fuller statistic. Table 2 presents the results and shows that the spot and futures prices include a unit root while all the first differences are stationary.

[INSERT TABLE 2 AROUND HERE]

5 Empirical results and interpretation

This section includes the empirical analysis and the results obtained when estimating the parameters of the models described above. We start with the in-sample analysis of the nine one-factor models.

5.1 In sample analysis

Table 3 includes the in-sample estimates of the parameters for each model and three measures of goodness of fit: the minimum of the sum of squared errors, $\sum_t \min SCR(\hat{\theta}_t) = \sum_t \hat{u}_t$, its standard deviation, $\frac{1}{n}\sum_{t=1}^{n} \hat{u}_t^{1/2}$, and the absolute mean error, $\frac{1}{n}\sum_{t=1}^{n} |\hat{u}_t|$. These estimates have been obtained using the entire sample available for each futures. It can be seen that the model 9 provides the smallest sum of squared errors, although the most notable improvement occurs when moving from model 2 to model 3. This suggests that, to explain futures prices, it is necessary to introduce a cyclical component, either long- or short-term. It should also be noted that the models that exhibit long-term oscillations have SCR very similar to the models that only incorporate seasonality.

[INSERT TABLE 3 AROUND HERE]

Table 4 provides the sums of squared errors for each maturity and its comparison for each model. We can conclude that including the Fourier terms in the process that follows the log-futures price improves the adjustment to market observed data, since the sum of squared errors decreases. The highest improvement appears in models 3 and 7, which incorporate long-term oscillations and a cyclical component, respectively. That is, the inclusion of the additional cyclical components of models 4, 5, 8, and 9 improves the adjustment but this improvement is not very high.

[INSERT TABLE 4 AROUND HERE]

We now discuss the main results obtained with each model:

- Model 2 includes a seasonal component but the improvement with respect to model 1 is marginal (0.83%). This fact does not indicate that futures prices do not present seasonality but that this is already incorporated in the price, since it is present in spot prices.

- Model 3 incorporates oscillations in the level of mean-reversion through a Fourier term and this incorporation leads to an improvement of almost 44% and 45% with respect to, respectively, models 2 and 1. According to this model, the estimated frequency of long-term fluctuations is approximately 20.5 years.

- Model 4 includes a seasonal component and a Fourier term with a frequency for the seasonal and cyclical components. The improvements produced by this model are, respectively, 51.11%, 50.71%, and 11.07% with respect to models 1, 2, and 3. In this model, the long-term oscillations in the level of mean-reversion are around 13 years, significantly lower than the frequency obtained for the previous model.

- Model 5 incorporates a second frequency in the Fourier series and produces an improvement of 52.21%, 51.81%, 13.06% and 2.25% with respect to the first four models. In this case, the estimated frequency for long-term oscillations is 14 years. The periodic component suggests a period of between 3.5 and 4.5 years, except for the second term that has a period of 8.5 years. Finally, the second frequency ranges between 4 and 10 years.

- The improvement in model 6 with respect to model 5 is not very relevant, which indicates that the effect of the last seasonal component is marginal when it comes to explain futures prices. The estimated frequency for long-term oscillations is 13.5 years and the seasonal frequencies range between 1.5 and 7 years.

- Model 7 is similar to the model 4, with the difference that in this model long-term oscillations are not incorporated in the level of mean-reversion. This model produces improvements of 47.48%, 47.04% and 4.45% with respect to models 1, 2, and 3, respectively. With respect to models 4 and 5, there is a worsening of 7.45% and 9.90%, respectively.

- Model 8 is similar to model 5 but it does not incorporate the long-term swing. The improvements that are produced are 52.29%, 51.89%, 13.21%, 2.40%, 0.17%, and 9.17% with respect to, respectively, models 1 to 6.

- Finally, the model 9 presents the smallest sum of squared errors although the improvement with respect to the model 8 is not very high.

Therefore, we can conclude that the ranking of models (from least to most suitable) is: 1 - 2 - 3 - 7 - 4 - 5 - 8 - 6 - 9. Although the highest relative improvement between models occurs in models 3 and 7, when including long-term oscillations, the model 9 presents the lowest SCR, so including seasonality also helps greatly to estimate the price of futures on CO_2 emissions.

Figure 2 shows the fitted futures prices for the nine estimated models and the observed futures prices. For the fifth maturity, all the models provide a good adjustment, being better in models 7, 4, and 6, and worse in models 1 and 2. Although we must emphasize that the model with the best fit is not always the same but depends on the period under analysis. For this reason, several periods will be studied later. Figure 3 shows that the spot effect does not exactly reproduce the level of the futures price but they present a very similar behavior.

[INSERT FIGURES 2 and 3 AROUND HERE]

Figure 4 shows the decomposition of the four effects of the model 5 for the second nearest maturity. These effects are: spot effect, volatility effect, effect of periodic (seasonal and cyclical) component, and effect of long-term oscillations. In figure 5, the level of mean-reversion (long-term

swing) is observed in the model 5 for the five maturities of the chosen futures. We can see a certain synchronization among the levels obtained for the different maturities.

[INSERT FIGURES 4 and 5 AROUND HERE]

We analyze now these models along two subperiods, since the Kyoto Protocol expired at the end of 2012 and its continuation was signed at the 2012 Doha summit. Then, we consider a first subperiod sicne the first available data for each futures until the end of 2012 and a second subperiod that starts in 2013 and ends in March, 2018.

Table 5 includes the estimates and the measures of goodness of fit for the first subperiod. Models 5 and 9 improve the results from the other models, being the model 9 the one with the highest gain, 31.90%, with respect to the model 2.

[INSERT TABLE 5 AROUND HERE]

Table 6 shows, for each maturity, the SCR, the seasonality period, and the improvement of each model. It can be seen that the improvements with respect to models 1 and 2 are not so high as when we consider the entire sample. Even so, certain models (such as 4 or 7) show notable improvements. The frequency of long-term oscillations is estimated around 10 years for models 3, 4, and 5, and around 17 years for the model 6.

[INSERT TABLE 6 AROUND HERE]

Figure 6 shows the fitted prices provided by the different models for the second maturity (the one with more available data) and the observed futures prices. In this case, models 5 and 9 are those that seem to produce a better fit.

[INSERT FIGURE 6 AROUND HERE]

For the second subperiod, Table 7 shows the estimated parameters and the measures of goodness of fit. The model 9 presents the lowest SCR, with an improvement of 41.07% over the model 1. Models 4, 5, 6, and 8 have a very similar SCR, so all these models could be considered reasonable when estimating these futures prices. In this subperiod, the highest improvement, 39.69%, occurs from model 2 to model 3.

[INSERT TABLE 7 AROUND HERE]

Table 8 shows the improvements and seasonality periods of each model for each maturity. We see that the frequency of long-term oscillations fluctuates between 10 and 13.5 years depending on the model. The highest improvements with respect to model 2 occur in models 3 and 7.

[INSERT TABLE 8 AROUND HERE]

Figure 7 illustrates that the model 6 provides the worst fit at the beginning of the sample but does not provide the highest sum of squared errors. This is because, although it fits poorly at the beginning, it adjusts quite well later, so that other models with an average adjustment along the sample end up having a higher SCR.

[INSERT FIGURE 7 AROUND HERE]

Table 9 summarizes the ranking of models (from lowest to highest SCR) along with the ranking of the estimated models on a yearly basis. We can see that the model 9 has the lowest SCR although the highest improvement occurs in other models, such as 4, 8, or 3.

[INSERT TABLE 9 AROUND HERE]

Then, we can say that the model 9 presents the lowest SCR for all the periods under analysis. However, in many cases, this SCR is similar to that in other models. Therefore, given the cost of adding more frequencies, one might think that one of the other models would also be appropriate. It is necessary to highlight that, in all the cases, models 1 and 2 do present a sum of squared errors that is significantly higher than in the remaining models, so including the seasonal components does significantly improve the estimates.

5.2 Forecasting analysis

This subsection studies whether the aforementioned models help to anticipate changes in the market prices of futures on CO_2 emission allowances. Then, we compare now all the models, focusing on their sum of squared forecasting errors.

Tables 10 to 13 summarize the main results. Table 10 shows the forecasting results for the year 2016 using data from 2015. The model 7 provides the lowest forecasting error but it is not the model with the best in-sample fit (model 9). Analyzing the different terms, the model 7 also presents the smallest error for the first term, the model 6 is the best one for the second term and the model 9 is the best one for the last two terms. The model 3 has the highest total error.

[INSERT TABLE 10 AROUND HERE]

If we use the information of 2014 and 2015, Table 11 shows that the model 3 presents the smallest error. It can be seen that, using the information of both years, the errors increase considerably. On a three months basis, models 5, 3, 4, and 9 are, respectively, the best ones for the different terms of 2016. Finally, the model 8 shows the highest overall error.

[INSERT TABLE 11 AROUND HERE]

Table 12 shows the results for 2017. Clearly, the model 4 is the best one as it presents the lowest forecasting errors. Once again, the model with the best in-sample behavior (model 9) does not show the best out-of-sample performance. The highest forecasting errors are obtained in the first and fourth terms. The model 3 provides the smallest errors in the first two terms. Models 1 and 4 obtain the best fit in the third term and the model 4 is the best one along the last term. The model 8 provides the worst overall fit.

[INSERT TABLE 12 AROUND HERE]

Using the information of 2015 and 2016, Table 13 shows that the model 5 provides the smallest error but the model 3 provides a similar result. The best performance for all the models happens in the last term. Looking at the different terms, models 5, 6, 3, and (again) 5 are the best ones for each term. As previously, the model 8 presents the worst global performance.

[INSERT TABLE 13 AROUND HERE]

Therefore, we have illustrated that the model with the best in-sample behavior is not neccesarily the best forecasting model. Moreover, we can conclude that there is not a clear winner model from an out-of-sample point of view.

5.3 Two-factor model

We analyze now the Bacaicoa *et al.* (2014) model that introduces the convenience yield as second factor. Table 14 presents our estimates and the sum of squared errors. We can see that the second and third closest-to-maturity futures present, respectively, the lowest and highest estimation errors.

[INSERT TABLE 14 AROUDN HERE]

Figure 8 shows the observed and fitted futures prices under this model. We can see a very good fit, outperforming any previous one-factor model. Figure 9 includes the spot price, the price of the fifth closest-to-maturity futures, and the convenience yield. Financial theory says that futures are typically in contango because the futures price is the compounded spot price. However, the presence of a convenience yield implies that a futures can be in contango or in backwardation depending whether the interest rate is, respectively, higher or lower than the convenience yield. This figure shows that the CO_2 futures market is in contango until 2015 but, from then on, the convenience yield is close to the interest rate and, then, futures quotations are close to spot prices.

[INSERT FIGURES 8 AND 9 AROUDN HERE]

6 Conclusions

CO_2 emissions are currently a very important issue with relevant implications in the global context due to the pernicious effects of these emissions on the atmosphere and global warming. This paper focuses on the analysis of futures on CO_2 emission allowances and tries to find the most adequate model in order to make both in- and out-of-sample estimations. To this aim, we have compared different specifications of the Moreno *et al.* (2019) model versus the models proposed in Schwartz (1997) and Lucía and Schwartz (2002). In addition, we have also introduced a two-factor model.

Moreno *et al.* (2019) proposed initially their model for prices of energy futures, which present long-term flctuations in the level of mean-reversion and seasonality, using Fourier series to represent these seasonal and cyclical components. The possible advantages of this model come from its flexibility to allow long-term oscillations in the level of mean-reversion withouth imposing any constraints on the frequencies to be estimated in the Fourier series. This model also provides closed-form expressions for the prices of different derivatives and, specifically, for futures prices.

We have used EUA data from 2009 until 2018 and we have estimated the models proposed in Schwartz (1997), Lucía and Schwartz (2002), seven versions of the model in Moreno *et al.* (2019), and a two-factor model (Bacaicoa *et al.*, 2014) that also uses Fourier series to estimate cyclical components. One-factor models have been estimated by applying non-linear least-squares regressions.

We have performed the estimations by considering different alternatives: a) for the whole sample, b) from the first available data until the end of 2012 (final moment of the Kyoto protocol), c) from 2013 until March, 2018, period in which the protocol is prolonged, thanks to the 2012 Doha summit and d) yearly estimates.

The in-sample empirical analysis shows that the model 9 (including three seasonal components and no frequency in the long-term oscillations in the level of mean-reversion) is the most adequate model in all the periods under study. However, in several cases, the improvement with respect to models 3 to 8 is not very noticeable. We can highlight that all the versions of the Moreno *el al.* (2019) model outperform the models in Schwartz (1997) and Lucía and Schwartz (2002). Then, we conclude that the introduction of seasonality with the Fourier series improves the in-sample estimation.

Out-of-sample estimates are obtained for the period 2016-2017 by applying the Kalman filter. The different versions of the Moreno *et al.* (2019) model do not seem so adequate as in the in-sample analysis since the sum of squared errors do not differ significantly from those obtained for the models in Schwartz (1997) and Lucía and Schwartz (2002). Models 7 and 4 provide the best forecasting behavior for respectively, 2016 and 2017. Hence, we conclude that the different cases embedded in Moreno *et al.* (2019) are adequate to perform the in-sample estimations but they are not so good for prediction purposes.

Finally, the two-factor model introduced in Bacaicoa *et al.* (2014) has been estimated by applying

also the Kalman filter, with the convenience yield as non-observable variable. We have checked that this model outperforms all the one-factor models as it provides a better fit to the data under analysis.

As future research lines, we can analyze the empirical behavior of the one-factor models analyzed in this paper to explain prices in electricity markets. We can also study the forecasting performance of the two-factor model presented in Bacaicoa *et al.* (2014). A final avenue for further research can be to enlarge this two-factor model by considering the different versions of the Moreno *et al.* (2019) model.

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Appendix of Tables

Descriptive statistics of the futures on CO_2 emissions.

	Beginning of the sample	Mean	Standard deviation	Coeff. of variation	Skewness	Kurtosis	Maximum	Minimum
Spot	27, October, 2009	8.13	3.64	0.45	0.95	2.58	16.8	2.7
c1	27, October, 2009	8.29	3.84	0.46	0.97	2.60	17.4	2.79
c2	27, October, 2009	8.55	4.04	0.47	0.98	2.58	18.25	2.89
c3	30, July, 2010	8.21	3.96	0.48	1.44	4.04	19.67	3.01
c4	30, July, 2010	8.52	4.24	0.50	1.45	4.03	20.9	3.16
c5	29, December, 2010	8.28	4.00	0.48	1.81	5.80	22.01	3.28
c6	3, April, 2012	6.91	1.67	0.24	0.34	2.27	11.39	3.48
c7	3, April, 2012	7.05	1.82	0.26	0.53	2.49	11.93	3.71

Table 1: Descriptive analysis of spot prices and futures prices on emissions of CO_2. We show mean, standard deviation, coefficient of variation, skewness, kurtosis, maximum, and minimum. For all the prices, the sample ends at 9, March, 2018.

Dickey-Fuller statistics.

	ADF (levels)		ADF (differences)	
	Value	Statistic	Value	Statistic
Spot	-0.83	-3.99	-45.61	-3.99
c1	-2.53	-3.99	-49.42	-3.99
c2	-2.54	-3.99	-49.31	-3.99
c3	-2.29	-3.99	-48.41	-3.99
c4	-2.14	-3.99	-46.49	-3.99
c5	-2.03	-3.99	-45.36	-3.99
c6	-3.21	-3.99	-41.12	-3.99
c7	-3.19	-3.99	-40.72	-3.99

Table 2: Augmented Dickey-Fuller statistic (for levels and first differences) for the spot price and futures c1 to c7.

Parametric in-sample estimations, whole sample.

Parameters	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
STRUCTURAL PARAMETERS									
α	46.7172 (6.0992)	45.3954 (2.9647)	3.2672 (18.3025)	2.7662 (0.0232)	2.1292 (0.3073)	5.0486 (0.2162)	-1.0545 (0.5854)	1.1762 (0.4426)	-0.0574 (0.1888)
σ^2	0.5770 (0.0057)	0.5787 (0.8948)	0.0000 (0.0000)	0.0000 (0.0000)	1.1931 (0.4097)	0.0006 (0.0037)	1.8230 (0.2498)	2.0574 (0.6252)	4.2494 (0.3059)
κ	0.0000 (0.0000)	0.000 (0.0012)	0.5252 (4.9997)	0.5852 (0.0154)	0.6314 (0.0411)	0.6718 (0.0142)	0.1843 (0.0081)	0.6621 (0.0195)	0.7309 (0.0146)
γ	-	0.0098 (0.0009)	-	-	-	-	-	-	-
ϕ	-	3.1416 (0.0509)	-	-	-	-	-	-	-
B_x	-	-	1.0 (3.7954)	0.9026 (0.0342)	1.0243 (0.0332)	-1.0647 (0.1989)	-	-	-
B_y	-	-	1.0 (23.6158)	-0.1937 (0.0341)	0.3711 (0.2333)	2.8779 (0.1574)	-	-	-
ω_z	-	-	2π0.0487 (1.8681)	2π0.0763 (0.0124)	2π0.0715 (0.0330)	2π0.0258 (0.0072)	-	-	-
Period	-	-	20.5 years	13 years	14 years	39 years	-	-	-
SEASONAL PARAMETERS									
C2									
$A_{x,1}$	-	-	-	-0.0341 (0.0181)	-3.1482 (1.8135)	-21.1646 (1.1073)	-0.2725 (0.0337)	0.2171 (0.0929)	-2.3336 (1.7732)
$A_{y,1}$	-	-	-	-0.0893 (0.0158)	9.9710 (0.7681)	-2.7597 (0.4128)	-0.4517 (0.0417)	-0.1056 (0.0575)	-6.7373 (0.8488)
$\omega_{f,1}$	-	-	-	2π0.2270 (0.0459)	2π0.1175 (0.0303)	2π0.2928 (0.0141)	2π0.10 (0.0147)	2π0.1787 (0.0697)	2π0.1473 (0.0304)
Period	-	-	-	5 years	8.5 years	3.5 years	10 years	5.5 years	7 years
$A_{x,2}$	-	-	-	-	3.7254 (1.7041)	0.0145 (0.0048)	-	0.2649 (0.1055)	13.0661 (2.2216)
$A_{y,2}$	-	-	-	-	-9.8436 (0.8479)	-0.0233 (0.0034)	-	-0.7122 (0.0953)	16.6759 (4.0166)
$\omega_{f,2}$	-	-	-	-	2π0.1160 (0.0423)	2π0.7075 (0.0465)	-	2π0.10 (0.0331)	2π0.1647 (0.0184)
Period	-	-	-	-	8.5 years	1.5 years	-	10 years	6 years
$A_{x,3}$	-	-	-	-	-	21.1372 (1.1082)	-	-	-10.1273 (0.7493)
$A_{y,3}$	-	-	-	-	-	2.7031 (0.4123)	-	-	-10.6168 (4.6234)
$\omega_{f,3}$	-	-	-	-	-	2π0.2930 (0.0141)	-	-	2π0.1704 (0.0285)
Period	-	-	-	-	-	3.5 years	-	-	6 years

Parameters	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
					C3				
$A_{x,1}$	-	-	-	-0.1011 (0.0100)	-0.8144 (0.5527)	4.9540 (0.2063)	-0.3908 (0.0271)	0.0920 (0.0755)	-3.9286 (0.2467)
$A_{y,1}$	-	-	-	0.0033 (0.0151)	3.3231 (0.5955)	7.8659 (0.4444)	-0.3923 (0.0437)	-0.6183 (0.0210)	-3.8063 (0.3533)
$\omega_{f,1}$	-	-	-	2π0.2616 (0.0282)	2π0.2472 (0.0291)	2π0.1402 (0.0200)	2π0.10 (0.0102)	2π0.10 (0.0218)	2π0.1342 (0.0129)
Period	-	-	-	4 years	4 years	7 years	10 years	10 years	7.5 years
$A_{x,2}$	-	-	-	-	0.8135 (0.5541)	-4.7483 (0.1924)	-	-0.10 (0.0309)	4.2508 (0.1781)
$A_{y,2}$	-	-	-	-	-3.3595 (0.5829)	-8.0998 (0.4536)	-	-0.1330 (0.0365)	2.5631 (0.3664)
ω f,2	-	-	-	-	2π0.2461 (0.0451)	2π0.1392 (0.0198)	-	2π0.2270 (0.0466)	2π0.1414 (0.0137)
Period	-	-	-	-	4 years	7 years	-	4.5 years	7 years
$A_{x,3}$	-	-	-	-	-	0.0008 (0.0148)	-	-	0.0468 (0.0059)
$A_{y,3}$	-	-	-	-	-	0.0747 (0.0051)	-	-	-0.0116 (0.0132)
$\omega_{f,3}$	-	-	-	-	-	2π0.3033 (0.0438)	-	-	2π0.3596 (0.0607)
Period	-	-	-	-	-	3 years	-	-	3 years
					C4				
$A_{x,1}$	-	-	-	-0.0628 (0.0260)	3.1510 (0.2851)	0.1112 (0.0145)	-0.4650 (0.0191)	-0.1409 (0.0141)	-0.1453 (0.0091)
$A_{y,1}$	-	-	-	0.0629 (0.0342)	0.6634 (0.4589)	-0.0312 (0.0315)	-0.5121 (0.0441)	-0.0147 (0.0384)	-0.0025 (0.0227)
$\omega_{f,1}$	-	-	-	2π0.2761 (0.0236)	2π0.2207 (0.1191)	2π0.1399 (0.0576)	2π0.10 (0.0134)	2π0.2442 (0.0576)	2π0.2478 (0.0298)
Period	-	-	-	4 years	4.5 years	7 years	10 years	4 years	4 years
$A_{x,2}$	-	-	-	-	-3.2104 (0.2875)	-1.3032 (0.1339)	-	-0.0739 (0.1005)	0.0378 (0.0763)
$A_{y,2}$	-	-	-	-	-0.5431 (0.4256)	-2.6795 (0.0998)	-	-0.5807 (0.0204)	-0.5884 (0.0082)
$\omega_{f,2}$	-	-	-	-	2π0.2227 (0.0346)	2π0.2667 (0.0219)	-	2π0.1629 (0.0310)	2π0.1000 (0.0231)
Period	-	-	-	-	4.5 years	4 years	-	10 years	10 years
$A_{x,3}$	-	-	-	-	-	1.2502 (0.1261)	-	-	-0.0114 (0.0011)
$A_{y,3}$	-	-	-	-	-	2.7106 (0.1019)	-	-	0.0053 (0.0016)
$\omega_{f,3}$	-	-	-	-	-	2π0.2673 (0.0217)	-	-	2π1.4294 (0.0354)
Period	-	-	-	-	-	4 years	-	-	1 año

Parameters	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
					C5				
$A_{x,1}$	-	-	-	-0.0160 (0.0580)	-0.0417 (0.0138)	0.9022 (1.2219)	-0.5121 (0.0186)	-0.2062 (0.0257)	-5.8394 (0.8724)
$A_{y,1}$	-	-	-	0.0902 (0.0095)	0.0872 (0.0158)	43.9179 (2.9767)	-0.3103 (0.0331)	-0.0861 (0.0912)	7.5718 (1.1985)
$\omega_{f,1}$	-	-	-	2π0.2877 (0.0282)	2π0.2786 (0.0455)	2π0.2000 (0.0193)	2π0.10 (0.0086)	2π0.2277 (0.0714)	2π0.1721 (0.0080)
Period	-	-	-	3.5 years	3.5 years	5 years	10 years	4.5 years	6 years
$A_{x,2}$	-	-	-	-	-0.1617 (0.0544)	-3.8639 (0.4296)	-	-0.5195 (0.1228)	4.7467 (0.8774)
$A_{y,2}$	-	-	-	-	-0.0074 (0.0659)	-21.8634 (1.5149)	-	-0.2867 (0.1569)	-7.9246 (1.1865)
$\omega_{f,2}$	-	-	-	-	2π0.10 (0.0403)	2π0.2039 (0.0184)	-	2π0.1270 (0.0587)	2π0.1757 (0.0080)
Period	-	-	-	-	10 years	5 years	-	8 years	6 years
$A_{x,3}$	-	-	-	-	-	2.6764 (0.8056)	-	-	0.0011 (0.0020)
$A_{y,3}$	-	-	-	-	-	-21.5944 (1.4597)	-	-	0.0114 (0.0009)
$\omega_{f,3}$	-	-	-	-	-	2π0.1964 (0.0202)	-	-	2π1.4448 (0.0390)
Period	-	-	-	-	-	5 years	-	-	1 año
					C6				
$A_{x,1}$	-	-	-	0.0215 (0.0126)	0.0234 (0.0315)	0.5844 (0.2212)	-0.5033 (0.0112)	0.0055 (0.0230)	-0.1831 (0.0523)
$A_{y,1}$	-	-	-	0.0816 (0.0210)	0.0893 (0.0102)	-1.8668 (0.1094)	-0.2266 (0.0611)	0.1055 (0.0101)	-0.4224 (0.0337)
$\omega_{f,1}$	-	-	-	2π0.2975 (0.0274)	2π0.2972 (0.0459)	2π0.3346 (0.0200)	2π0.10 (0.0184)	2π0.2885 (0.0430)	2π0.1015 (0.0251)
Period	-	-	-	3.5 years	3.5 years	3 years	10 years	4 years	10 years
$A_{x,2}$	-	-	-	-	-0.2252 (0.0357)	0.2477 (0.1625)	-	-0.1731 (0.0599)	0.0381 (0.0153)
$A_{y,2}$	-	-	-	-	-0.0305 (0.0460)	1.4975 (0.1126)	-	-0.4760 (0.0401)	0.1014 (0.0110)
$\omega_{f,2}$	-	-	-	-	2π0.10 (0.0304)	2π0.3476 (0.0205)	-	2π0.10 (0.0210)	2π0.2973 (0.0342)
Period	-	-	-	-	10 years	3 years	-	10 years	3.5 years
$A_{x,3}$	-	-	-	-	-	-0.4568 (0.0293)	-	-	0.0086 (0.0029)
$A_{y,3}$	-	-	-	-	-	-0.2293 (0.0597)	-	-	0.0078 (0.0030)
$\omega_{f,3}$	-	-	-	-	-	2π0.2810 (0.0204)	-	-	2π1.4546 (0.0636)
Period	-	-	-	-	-	3.5 years	-	-	1 año
$\sum_t \min SCR(\hat{\theta}_t) = \sum_t \hat{u}_t$	122.26	121.24	67.21	59.7646	58.4287	57.8309	64.2147	58.3276	57.0128
$(\frac{1}{n}\sum_{t=1}^{n} \hat{u}_t)^{1/2}$	0.1280	0.1274	0.0949	0.0895	0.0885	0.0880	0.0927	0.0884	0.0874
$\frac{1}{n}\sum_{t=1}^{n} \lvert\hat{u}_t\rvert$	0.0887	0.0886	0.0561	0.0499	0.0495	0.0495	0.0499	0.0494	0.0490

Table 3: Parameters of the in-sample estimations for the nine one-factor models, periods, and goodness of fit measures for the whole sample period. In parenthesis we show the standard deviations of the estimations.

SCR, seasonality, and long-term oscillations. Sample period: first available data - 2018.

$\sum_t \min SCR(\hat{\theta}_t)$	C2	C3	C4	C5	C6
Model 1	8.15	18.92	17.51	18.89	29.46
Model 2	8.08	18.72	17.40	18.83	29.42
Seasonality (years)	1	1	1	1	1
Improvement over model 1 (%)	0.83	1.08	0.63	0.33	0.14
Model 3	7.002	14.70	10.00	7.33	11.34
Long-term swing (years)	20.5	20.5	20.5	20.5	20.5
Improvement over model 2 (%)	13.39	21.42	42.53	61.08	61.47
Model 4	6.95	13.85	8.77	5.59	8.79
Long-term swing (years)	13	13	13	13	13
Seasonality (years)	4.5	4	4	3.5	3.5
Improvement over model 3 (%)	0.74	5.78	12.30	23.74	22.49
Model 5	6.69	13.63	8.80	5.62	8.46
Long-term swing (years)	14	14	14	14	14
Seasonality (years)	8.5	4	4.5	3.5	3.5
Seasonality (years)	8.5	4	4.5	10	10
Improvement over model 4 (%)	3.74	1.59	-0.34	-0.54	3.75
Model 6	6.67	13.41	8.72	5.74	8.09
Long-term swing (years)	13.5	13.5	13.5	13.5	13.5
Seasonality (years)	3.5	7	7	5	3
Seasonality (years)	1.5	7	4	5	3
Seasonality (years)	3.5	3	4	5	3.5
Improvement over model 5 (%)	0.30	1.61	0.91	2.14	4.37
Model 7	6.92	14.84	9.53	6.04	9.53
Seasonality (years)	10	10	10	10	10
Improvement over model 2 (%)	14.36	20.73	45.23	67.93	67.61
Model 8	6.62	13.57	8.74	5.74	8.56
Seasonality (years)	5.5	10	5	4.5	3.5
Seasonality (years)	10	5	10	8	10
Improvement over model 7 (%)	4.34	8.56	8.29	4.97	10.18
Model 9	6.51	13.27	8.61	5.80	8.46
Seasonality (years)	7	7.5	4	6	10
Seasonality (years)	6	7	10	6	3.5
Seasonality (years)	6	3	1	1	1
Improvement over model 8 (%)	1.66	2.21	1.49	-1.05	1.17

Table 4: Sum of squared pricing errors, seasonal periods, long-term swing in the mean-reversion level, and statistical improvement of each model. Sample: first available data until 2018.

Parametric estimations, since available data until 2013.

Parameters	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
STRUCTURAL PARAMETERS									
α	6.0620 (0.2158)	5.7335 (0.1734)	3.0212 (98.7283)	-0.5866 (0.3431)	3.9163 (0.0698)	9.3737 (0.1526)	0.23141 (0.1828)	1.7790 (0.0296)	1.8559 (0.0239)
σ^2	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	6.8079 (0.4039)	6.9084 (0.3060)	7.2207 (0.2662)	7.8655 (0.4547)	5.0078 (0.2306)	7.3708 (0.2201)
κ	0.1663 (0.0102)	0.1871 (0.0086)	0.8251 (16.7225)	2.0768 (0.0360)	3.3624 (0.0696)	3.4368 (0.0606)	1.2271 (0.0214)	2.6670 (0.0398)	3.3863 (0.0552)
γ	-	0.0312 (0.0007)	-	-	-	-	-	-	-
ϕ	-	0.2046 (0.0041)	-	-	-	-	-	-	-
B_x	-	-	0.7297 (219.8136)	0.9948 (0.3417)	-6.0768 (0.1347)	1.1014 (0.0425)	-	-	-
B_y	-	-	-0.0093 (145.8114)	-2.1419 (0.2154)	5.1709 (0.1056)	1.7244 (0.1202)	-	-	-
ω_z	-	-	2π0.10 (186.7926)	2π0.10 (0.0496)	2π0.0582 (0.0021)	2π0.0522 (0.0064)	-	-	-
Period	-	-	10 years	10 years	10 years	17 years	-	-	-
SEASONAL PARAMETERS									
C2									
$A_{x,1}$	-	-	-	0.4686 (0.0237)	-0.0727 (0.0024)	2.3601 (0.2590)	0.5857 (0.0153)	14.4101 (0.6657)	-1.6247 (0.0581)
$A_{y,1}$	-	-	-	-0.3636 (0.0253)	-0.0032 (0.0058)	1.3521 (0.1210)	0.3429 (0.0578)	-4.2427 (1.8977)	-0.4881 (0.1704)
$\omega_{f,1}$	-	-	-	2π0.2245 (0.0171)	2π0.8649 (0.0429)	2π0.5024 (0.0184)	2π0.10 (0.0361)	2π0.2244 (0.0245)	2π0.2542 (0.0303)
Period	-	-	-	4.5 years	1 año	2 years	10 years	4.5 years	4 years
$A_{x,2}$	-	-	-	-	0.2662 (0.1637)	-0.1674 (0.0097)	-	-14.4283 (0.6657)	1.5856 (0.0740)
$A_{y,2}$	-	-	-	-	-2.0712 (0.0697)	-0.2328 (0.0327)	-	3.8628 (1.8979)	0.5516 (0.1655)
$\omega_{f,2}$	-	-	-	-	2π0.1729 (0.0222)	2π0.7508 (0.0494)	-	2π0.2243 (0.0246)	2π0.2332 (0.0300)
Period	-	-	-	-	6 years	1.5 years	-	4.5 years	4 years
$A_{x,3}$	-	-	-	-	-	-2.5456 (0.2874)	-	-	-0.0665 (0.0026)
$A_{y,3}$	-	-	-	-	-	-0.0817 (0.0827)	-	-	0.0189 (0.0051)
$\omega_{f,3}$	-	-	-	-	-	2π0.5441 (0.0165)	-	-	2π0.8928 (0.0416)
Period	-	-	-	-	-	2 years	-	-	1 año

Parameters	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
					C3				
$A_{x,1}$	-	-	-	0.0059 (0.0206)	0.5384 (0.1674)	0.0961 (0.0165)	0.5661 (0.0907)	8.1621 (0.1650)	-2.0323 (0.1795)
$A_{y,1}$	-	-	-	-0.3946 (0.0184)	-2.2191 (0.0619)	-0.1842 (0.0346)	-0.1633 (0.0850)	19.2284 (0.6285)	4.5440 (0.4813)
$\omega_{f,1}$	-	-	-	2π0.2736 (0.0211)	2π0.1626 (0.0196)	2π0.4176 (0.0221)	2π0.2145 (0.0816)	2π0.3949 (0.0161)	2π0.4448 (0.0327)
Period	-	-	-	4 years	6 years	2.5 years	8 years	2.5 years	2 years
$A_{x,2}$	-	-	-	-	-0.0749 (0.0033)	-0.7442 (0.0415)	-	-8.7701 (0.1634)	0.9065 (0.2272)
$A_{y,2}$	-	-	-	-	0.0313 (0.0057)	-0.0840 (0.0568)	-	-18.6820 (0.6356)	-4.6988 (0.4403)
ω f,2	-	-	-	-	2π0.8747 (0.0379)	2π0.2869 (0.0357)	-	2π0.3968 (0.0161)	2π0.4631 (0.0289)
Period	-	-	-	-	1 año	3.5 years	-	2.5 years	2 years
$A_{x,3}$	-	-	-	-	-	-0.0965 (0.029)	-	-	-0.0337 (0.0162)
$Ay, 3$	-	-	-	-	-	-0.0509 (0.0101)	-	-	-0.2298 (0.0201)
$\omega_{f,3}$	-	-	-	-	-	2π0.8072 (0.0466)	-	-	2π0.7322 (0.0407)
Period	-	-	-	-	-	1.5 years	-	-	1.5 years
					C4				
$A_{x,1}$	-	-	-	-0.2399 (0.0114)	0.3621 (0.1581)	20.8899 (0.7646)	-0.2264 (0.0110)	0.8159 (0.0329)	-0.2213 (0.0139)
$A_{y,1}$	-	-	-	0.0065 (0.0185)	-2.3290 (0.0643)	1.0828 (0.1518)	0.6999 (0.0281)	-0.2218 (0.0366)	-0.0464 (0.0162)
$\omega_{f,1}$	-	-	-	2π0.3678 (0.0296)	2π0.1639 (0.0175)	2π0.2867 (0.0199)	2π0.2462 (0.0474)	2π0.1014 (0.0186)	2π0.3177 (0.0327)
Period	-	-	-	3 years	6 years	3.5 years	4 years	10 years	3 years
$A_{x,2}$	-	-	-	-	-0.0402 (0.0589)	-21.5728 (0.7637)	-	-0.0695 (0.0036)	-0.0563 (0.0115)
$A_{y,2}$	-	-	-	-	0.0589 (0.0032)	-1.4009 (0.1432)	-	-0.0447 (0.0051)	-0.1959 (0.0162)
$\omega_{f,2}$	-	-	-	-	2π0.8766 (0.0395)	2π0.2851 (0.0186)	-	2π0.7463 (0.0310)	2π0.2034 (0.0213)
Period	-	-	-	-	1 año	3.5 years	-	1.5 years	5 years
$A_{x,3}$	-	-	-	-	-	-0.0709 (0.0063)	-	-	-0.0086 (0.0063)
$A_{y,3}$	-	-	-	-	-	0.0397 (0.0074)	-	-	0.0627 (0.0014)
$\omega_{f,3}$	-	-	-	-	-	2π0.8459 (0.0555)	-	-	2π0.9111 (0.0506)
Period	-	-	-	-	-	1 año	-	-	1 año

Parameters	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
					C5				
$A_{x,1}$	-	-	-	0.1331 (0.0074)	10.4344 (0.1880)	20.1130 (0.1466)	0.6999 (0.1213)	1.5904 (0.0618)	0.5213 (0.0102)
$A_{y,1}$	-	-	-	-0.0520 (0.0136)	11.9826 (0.2923)	-10.8276 (0.3661)	-0.3858 (0.0755)	-0.2047 (0.0748)	-0.1495 (0.0274)
$\omega_{f,1}$	-	-	-	2π0.5753 (0.0386)	2π0.3822 (0.0087)	2π0.4696 (0.0196)	2π0.10 (0.0493)	2π0.4146 (0.0189)	2π0.4786 (0.0257)
Period	-	-	-	2 years	2.5 years	2 years	10 years	2.5 years	2 years
$A_{x,2}$	-	-	-	-	-12.0126 (0.2186)	-19.3101 (0.2432)	-	-0.6532 (0.0741)	0.3258 (0.0074)
$A_{y,2}$	-	-	-	-	-7.9467 (0.2230)	12.3856 (0.1674)	-	1.2013 (0.0498)	0.0804 (0.0189)
$\omega_{f,2}$	-	-	-	-	2π0.3974 (0.0089)	2π0.4759 (0.0231)	-	2π0.4652 (0.0187)	2π0.6208 (0.0290)
Period	-	-	-	-	2.5 years	2 years	-	2 years	1.5 years
$A_{x,3}$	-	-	-	-	-	0.8929 (0.3621)	-	-	0.0271 (0.0015)
$A_{y,3}$	-	-	-	-	-	-0.2606 (0.2314)	-	-	0.0029 (0.0065)
$\omega_{f,3}$	-	-	-	-	-	2π0.5033 (0.0196)	-	-	2π1.5269 (0.0915)
Period	-	-	-	-	-	2 years	-	-	1 año
					C6				
$A_{x,1}$	-	-	-	0.2384 (0.2921)	-14.0722 (0.3386)	-7.9319 (0.7617)	0.3214 (0.6993)	0.0346 (0.0169)	4.4377 (0.1740)
$A_{y,1}$	-	-	-	-0.3562 (0.1160)	0.7565 (0.0.1857)	-29.6918 (0.8623)	-0.2158 (0.3670)	-0.0214 (0.0224)	1.9128 (0.0470)
$\omega_{f,1}$	-	-	-	2π0.2843 (0.2345)	2π0.5769 (0.0104)	2π0.4693 (0.0164)	2π0.1001 (0.5373)	2π1.0404 (0.1816)	2π0.6923 (0.0116)
Period	-	-	-	3.5 years	1 año	2 years	10 years	1 año	1.5 years
$A_{x,2}$	-	-	-	-	11.1213 (0.1925)	8.7426 (0.4560)	-	0.0288 (0.0480)	1.2792 (0.0190)
$A_{y,2}$	-	-	-	-	-6.5650 (0.3163)	11.9233 (0.8693)	-	-0.0516 (0.0259)	-3.6162 (0.1550)
$\omega_{f,2}$	-	-	-	-	2π0.6017 (0.0112)	2π0.4645 (0.0161)	-	2π1.5764 (0.2810)	2π0.6115 (0.0103)
Period	-	-	-	-	1.5 years	2 years	-	1 año	1.5 years
$A_{x,3}$	-	-	-	-	-	3.5817 (0.2012)	-	-	1.0357 (0.0166)
$A_{y,3}$	-	-	-	-	-	17.5365(0.0884)	-	-	-0.5751 (0.0563)
$\omega_{f,3}$	-	-	-	-	-	2π0.4843 (0.0136)	-	-	2π0.9078 (0.0164)
Period	-	-	-	-	-	2 years	-	-	1 año
$\sum_t \min SCR(\hat{\theta}_t) = \sum_t \hat{u}_t$	24.2634	21.8772	21.8082	18.1176	15.1486	15.1911	19.5338	16.9416	14.8985
$(\frac{1}{n}\sum_{t=1}^{n} \hat{u}_t)^{1/2}$	0.1607	0.1526	0.1523	0.1388	0.1264	0.1271	0.1442	0.1342	0.1259
$\frac{1}{n}\sum_{t=1}^{n} \lvert\hat{u}_t\rvert$	0.0656	0.0652	0.0608	0.0574	0.0578	0.0582	0.0593	0.0565	0.0566

Table 5: Parameters of the in-sample estimations for the nine one-factor models, periods, and goodness of fit measures. The sample is from the first available data until 2013. In parenthesis we show the standard deviations of the estimations.

SCR, seasonality, and long-term oscillations. Sample period: first available data - 2012.

$\sum_t \min SCR(\hat{\theta}_t)$	C2	C3	C4	C5	C6
Model 1	1.13	2.20	1.36	1.04	1.25
Model 2	1.08	1.99	1.24	0.99	1.20
Seasonality (years)	1	1	1	1	1
Improvement over model 1 (%)	4.9	9.46	8.53	5.26	4.05
Model 3	1.06	1.82	1.03	0.74	0.94
Long-term swing (years)	10	10	10	10	10
Improvement over model 2 (%)	1.61	8.51	17.49	25.11	22.29
Model 4	0.88	1.48	0.93	0.72	0.74
Long-term swing (years)	10	10	10	10	10
Seasonality (years)	4.5	4	3	2	3.5
Improvement over model 3 (%)	16.75	18.51	6.54	3.17	21.11
Model 5	0.84	1.30	0.99	0.84	0.65
Long-term swing (years)	10	10	10	10	10
Seasonality (years)	1	6	6	2.5	2
Seasonality (years)	6	1	1	2.5	1
Improvement over model 4 (%)	4.54	12.34	-6.29	-16.70	11.29
Model 6	0.84	1.26	0.98	0.87	0.69
Long-term swing (years)	17	17	17	17	17
Seasonality (years)	2	2.5	3.5	2	2
Seasonality (years)	1.5	3.5	3.5	2	2
Seasonality (years)	2	1.5	1	2	2
Improvement over model 5 (%)	0.02	2.88	0.84	-3.93	-6.13
Model 7	0.93	1.67	1.05	0.77	0.79
Seasonality (years)	10	8	4	10	10
Improvement over model 2 (%)	13.13	16.28	15.77	21.87	34.66
Model 8	0.89	1.41	0.89	0.75	0.64
Seasonality (years)	4.5	2.5	10	2.5	1
Seasonality (years)	4.5	2.5	1.5	2	1
Improvement over model 7 (%)	4.81	15.51	15.33	3.36	19.09
Model 9	0.85	1.25	1.01	0.72	0.64
Seasonality (years)	4	2	3	2	1.5
Seasonality (years)	4	2	5	1.5	1.5
Seasonality (years)	1	1.5	1	1	1.5
Improvement over model 8 (%)	3.85	10.85	-13.82	2.93	0.31

Table 6: Sum of squared pricing errors, seasonal periods, long-term swing in the mean reversion level, and statistical improvement of each model. Sample: first available data until 2012.

Parametric estimations for the period 2013-2018.

Parameters	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
STRUCTURAL PARAMETERS									
α	2.1024 (0.0063)	2.0919 (0.0566)	2.7194 (0.0360)	2.0465 (0.7347)	2.9701 (0.2013)	11.1717 (0.1213)	2.1345 (0.0078)	0.7396 (0.1845)	1.2160 (0.1501)
σ^2	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	2.6722 (0.5054)	1.7800 (0.1989)	1.0584 (0.1069)	0.0000 (0.0000)	2.8723 (0.3173)	2.5383 (0.2796)
κ	0.6061 (0.0101)	0.6276 (0.0824)	0.2834 (0.0084)	0.7233 (0.0154)	0.7746 (0.0174)	0.7600 (0.0143)	0.5509 (0.0601)	0.7553 (0.0142)	0.8299 (0.0497)
γ	-	-0.0048 (0.0007)	-	-	-	-	-	-	-
ϕ	-	1.0962 (0.0241)	-	-	-	-	-	-	-
B_x	-	-	-0.0668 (0.0760)	3.1648 (0.9031)	2.1618 (0.1353)	-2.3127 (0.1557)	-	-	-
B_y	-	-	-1.0 (0.0398)	1.7277 (1.7081)	1.5273 (0.1846)	10.0570 (0.1334)	-	-	-
ω_z	-	-	2π0.10 (0.0119)	2π0.0856 (0.1020)	2π0.0737 (0.0062)	2π0.0370 (0.0028)	-	-	-
Period	-	-	10 years	12 years	13.5 years	27 years	-	-	
SEASONAL PARAMETERS									
C2									
$A_{x,1}$	-	-	-	0.5837 (0.1920)	24.4853 (0.1721)	2.1935 (0.4392)	0.0822 (0.0299)	-16.2817 (1.0068)	9.9040 (1.1590)
$A_{y,1}$	-	-	-	1.1918 (0.4895)	1.3200 (2.5632)	-5.2457 (0.5131)	-0.1473 (0.0494)	26.3427 (1.2936)	-21.1874 (0.3571)
$\omega_{f,1}$	-	-	-	2π0.1408 (0.0398)	2π0.2475 (0.0305)	2π0.2930 (0.0212)	2π0.2162 (0.0703)	2π0.1209 (0.0048)	2π0.1896 (0.0119)
Period	-	-	-	7 years	4 years	3.5 years	4.5 years	8 years	5 years
$A_{x,2}$	-	-	-	-	-24.8286 (0.2350)	11.6159 (0.6748)	-	18.1015 (1.0364)	-13.8954 (1.0497)
$A_{y,2}$	-	-	-	-	-2.1523 (2.6005)	15.5812 (0.9319)	-	-25.6476 (1.2769)	16.1317 (0.4269)
$\omega_{f,2}$	-	-	-	-	2π0.2467 (0.0149)	2π0.2908 (0.0219)	-	2π0.1191 (0.0045)	2π0.1820 (0.0125)
Period	-	-	-	-	4 years	3.5 years	-	8 years	5.5 years
$A_{x,3}$	-	-	-	-	-	-14.5431 (0.4272)	-	-	2.0090 (0.1254)
$A_{y,3}$	-	-	-	-	-	-9.7698 (0.4871)	-	-	1.7248 (0.1096)
$\omega_{f,3}$	-	-	-	-	-	2π0.2917 (0.0226)	-	-	2π0.2244 (0.0203)
Period	-	-	-	-	-	3.5 years	-	-	4.5 years

Parameters	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
					C3				
$A_{x,1}$	-	-	-	0.5187 (0.2103)	-36.4487 (1.1556)	-19.9361 (0.4335)	0.0818 (0.0205)	-19.2686 (0.9964)	-26.0575 (1.6385)
$A_{y,1}$	-	-	-	1.3027 (0.4964)	-6.6253 (3.0065)	0.6519 (0.1372)	-0.1867 (0.1355)	8.8366 (0.2377)	15.3824 (1.2162)
$\omega_{f,1}$	-	-	-	2π0.1386 (0.0374)	2π0.2411 (0.0089)	2π0.3235 (0.0055)	2π0.2182 (0.0588)	2π0.1121 (0.0033)	2π0.1185 (0.0099)
Period	-	-	-	7 years	4 years	3 years	4.5 years	9 years	8.5 years
$A_{x,2}$	-	-	-	-	36.1834 (1.1509)	19.1923 (0.3961)	-	20.7021 (1.0037)	28.4936 (1.5290)
$A_{y,2}$	-	-	-	-	5.5568 (3.0099)	2.3764 (0.1526)	-	-6.4771 (0.1802)	-8.2194 (1.3070)
$\omega_{f,2}$	-	-	-	-	2π0.2418 (0.0075)	2π0.3195 (0.0054)	-	2π0.1086 (0.0027)	2π0.118 (0.0096)
Period	-	-	-	-	4 years	3 years	-	9 years	9 years
$A_{x,3}$	-	-	-	-	-	-0.4267 (0.0323)	-	-	-1.0141 (0.1214)
$A_{y,3}$	-	-	-	-	-	-0.8069 (0.0363)	-	-	-1.1894 (0.1688)
$\omega_{f,3}$	-	-	-	-	-	2π0.3756 (0.0102)	-	-	2π0.1647 (0.0178)
Period	-	-	-	-	-	2.5 years	-	-	6 years
					C4				
$A_{x,1}$	-	-	-	0.2683 (0.2457)	6.8861 (0.5925)	4.0336 (0.1149)	0.1744 (0.0110)	1.3028 (0.1847)	16.1787 (1.6827)
$A_{y,1}$	-	-	-	1.4576 (0.4713)	8.3156 (0.9931)	12.3560 (0.0947)	0.2052 (0.1419)	1.3076 (0.0288)	27.6922 (2.0443)
$\omega_{f,1}$	-	-	-	2π0.1333 (0.0309)	2π0.2226 (0.0147)	2π0.2864 (0.0058)	2π0.2023 (0.0249)	2π0.1502 (0.0075)	2π0.1398 (0.0216)
Period	-	-	-	7 years	4.5 years	3.5 years	5 years	7 years	7 years
$A_{x,2}$	-	-	-	-	-6.4906 (0.6108)	-16.5140 (0.2285)	-	-0.3434 (0.2789)	-14.9998 (1.7860)
$A_{y,2}$	-	-	-	-	-9.2355 (0.9613)	-19.4722 (0.2143)	-	-2.0313 (0.0893)	-28.7678 (2.0816)
$\omega_{f,2}$	-	-	-	-	2π0.2207 (0.0122)	2π0.2952 (0.0068)	-	2π0.1306 (0.0175)	2π0.1383 (0.0218)
Period	-	-	-	-	4.5 years	3.5 years	-	7.5 years	7 years
$A_{x,3}$	-	-	-	-	-	10.8037 (0.2816)	-	-	0.0063 (0.0025)
$A_{y,3}$	-	-	-	-	-	6.9909 (0.27865)	-	-	0.0054 (0.0076)
$\omega_{f,3}$	-	-	-	-	-	2π0.3031 (0.0069)	-	-	2π1.4830 (0.0629)
Period	-	-	-	-	-	3 years	-	-	1 año

Parameters	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
					C5				
$A_{x,1}$	-	-	-	0.0881 (0.2804)	-0.0129 (0.0042)	0.0558 (0.0209)	0.2052 (0.0063)	6.7533 (0.4439)	36.6138 (0.6541)
$A_{y,1}$	-	-	-	1.5548 (0.4494)	0.0024 (0.0045)	-0.4175 (0.0116)	-0.0540 (0.0462)	-1.3413 (0.0.3134)	12.6518 (2.5441)
$\omega_{f,1}$	-	-	-	2π0.1299 (0.0338)	2π0.7366 (0.0651)	2π0.1979 (0.0084)	2π0.1938 (0.0621)	2π0.1639 (0.0081)	2π0.1520 (0.0219)
Period	-	-	-	7.5 years	1.5 years	5 years	5 years	6 years	6.5 years
$A_{x,2}$	-	-	-	-	0.5798 (0.0307)	4.9736 (0.0875)	-	-7.0598 (0.4111)	-36.4593 (0.6516)
$A_{y,2}$	-	-	-	-	-0.0320 (0.0325)	1.7154 (0.0634)	-	0.4823 (0.3448)	-14.0639 (2.4403)
$\omega_{f,2}$	-	-	-	-	2π0.1678 (0.0116)	2π0.4628 (0.0168)	-	2π0.1597 (0.0073)	2π0.1509 (0.0209)
Period	-	-	-	-	6 years	2 years	-	6 years	6.5 years
$A_{x,3}$	-	-	-	-	-	-4.9573 (0.0886)	-	-	-0.0023 (0.0093)
$A_{y,3}$	-	-	-	-	-	-1.8226 (0.0618)	-	-	-0.0100 (0.0014)
$\omega_{f,3}$	-	-	-	-	-	2π0.4623 (0.0167)	-	-	2π1.5254 (0.0527)
Period	-	-	-	-	-	2 years	-	-	1 año
					C6				
$A_{x,1}$	-	-	-	-0.0351 (0.2888)	-7.6119 (2.6628)	-25.9863 (0.4326)	0.2182 (0.0056)	-2.6361 (0.2078)	-4.7098 (0.1766)
$A_{y,1}$	-	-	-	1.6271 (0.4729)	-14.7730 (0.1025)	4.9824 (0.3952)	-0.0602 (0.0507)	0.7075 (0.0979)	-1.1071 (0.3393)
$\omega_{f,1}$	-	-	-	2π0.1276 (0.0283)	2π0.1615 (0.0098)	2π0.2523 (0.0133)	2π0.1929 (0.0492)	2π0.1625 (0.0045)	2π0.1521 (0.0164)
Period	-	-	-	7.5 years	6 years	4 years	5 years	6 years	6.5 years
$A_{x,2}$	-	-	-	-	8.2707 (2.6590)	27.7239 (0.4595)	-	2.2381 (0.1880)	4.5605 (0.2079)
$A_{y,2}$	-	-	-	-	14.8580 (0.1139)	-4.6863 (0.3952)	-	-1.3165 (0.1044)	-0.1282 (0.2376)
$\omega_{f,2}$	-	-	-	-	2π0.1615 (0.0081)	2π0.2523 (0.0130)	-	2π0.1719 (0.0041)	2π0.1602 (0.0147)
Period	-	-	-	-	6 years	4 years	-	6 years	6 years
$A_{x,3}$	-	-	-	-	-	-1.7794 (0.0462)	-	-	-0.0079 (0.0041)
$A_{y,3}$	-	-	-	-	-	-1.1611 (0.0561)	-	-	-0.0118 (0.0024)
$\omega_{f,3}$	-	-	-	-	-	2π0.2282 (0.0085)	-	-	2π1.5250 (0.0491)
Period	-	-	-	-	-	4.5 years	-	-	1 año
$\sum_t \min SCR(\hat{\theta}_t) = \sum_t \hat{u}_t$	57.4106	57.2739	39.7311	34.5377	34.1768	34.1694	35.1739	34.2873	33.753
$(\frac{1}{n}\sum_{t=1}^{n} \hat{u}_t)^{1/2}$	0.0938	0.0937	0.0780	0.0727	0.0723	0.0723	0.0734	0.0725	0.0719
$\frac{1}{n}\sum_{t=1}^{n} \lvert\hat{u}_t\rvert$	0.0680	0.0679	0.0489	0.0459	0.0460	0.0461	0.0463	0.0460	0.0451

Table 7: Parameters of the in-sample estimations for the nine one-factor models, periods, and goodness of fit measures. The sample covers the period 2013-2018. In parenthesis we show the standard deviations of the estimations.

SCR, seasonality, and long-term oscillations. Sample period: 2013-2018.

$\sum_t \min SCR(\hat{\theta}_t)$	C2	C3	C4	C5	C6
Model 1	5.98	13.17	10.76	10.56	16.95
Model 2	5.99	13.19	10.72	10.50	16.87
Seasonality (years)	1	1	1	1	1
Improvement over model 1 (%)	-0.21	-0.16	0.32	0.53	0.47
Model 3	5.79	12.47	8.04	4.96	8.47
Long-term swing (years)	10	10	10	10	10
Improvement over model 2 (%)	3.35	5.45	25.05	52.75	49.79
Model 4	5.30	10.84	6.92	4.45	7.03
Long-term swing (years)	11.5	11.5	11.5	11.5	11.5
Seasonality (years)	7	7	7.5	7.5	8
Improvement over model 3 (%)	8.46	13.09	13.92	10.23	17.07
Model 5	5.29	10.70	6.91	4.28	7.00
Long-term swing (years)	13.5	13.5	13.5	13.5	13.5
Seasonality (years)	4	4	4.5	1.5	6
Seasonality (years)	4	4	4.5	6	6
Improvement over model 4 (%)	0.22	1.30	0.14	3.85	0.38
Model 6	5.37	10.65	6.90	4.32	6.92
Long-term swing (years)	13.5	13.5	13.5	13.5	13.5
Seasonality (years)	3.5	3	3.5	5	4
Seasonality (years)	3.5	3	3.5	2	4
Seasonality (years)	3.5	2.5	3	2	4.5
Improvement over model 5 (%)	-1.51	0.47	0.14	-0.93	1.14
Model 7	5.44	11.14	7.05	4.38	7.17
Seasonality (years)	4.5	4.5	5	5	5
Improvement over model 2 (%)	9.18	15.54	34.24	58.29	57.50
Model 8	5.29	10.67	6.91	4.45	6.98
Seasonality (years)	8	9	6.5	6	6
Seasonality (years)	8	9	7.5	6	6
Improvement over model 7 (%)	2.76	4.22	1.99	-1.60	2.65
Model 9	5.27	10.55	6.77	4.35	6.79
Seasonality (years)	5	8.5	7	6.5	6.5
Seasonality (years)	5.5	9	7	6.5	6
Seasonality (years)	4.5	6	1	1	1
Improvement over model 8 (%)	0.38	1.12	2.03	2.25	2.72

Table 8: Sum of squared pricing errors, seasonal periods, long-term swing in the mean reversion level, and statistical improvement of each model. The sample period is 2013-2018.

Ranking of models, from lowest to highest SCR.

Years	Whole sample	Until 2013	2013-2018	2013-2014	2014-2015	2015-2016	2016-2017
	9 (57.01)	9 (14.90)	9 (33.75)	9 (13.60)	9 (4.73)	9 (0.94)	9 (4.28)
	6 (57.83)	5 (15.15)	6 (34.17)	8 (13.71)	6 (5.01)	8(0.97)	8 (4.41)
	8 (58.33)	6 (15.19)	5 (34.18)	6 (13.73)	8 (5.09)	5 (0.97)	4 (4.69)
	5 (58.43)	8 (16.94)	8 (34.29)	5 (13.91)	5 (5.09)	6 (0.97)	6 (4.75)
	4 (59.76)	4 (18.12)	4 (34.54)	4 (14.29)	4 (5.14)	4(0.99)	5 (4.86)
	7 (64.21)	7 (19.53)	7 (35.17)	7 (15.75)	7 (5.15)	7 (1.02)	7 (4.86)
	3 (67.21)	1 (24.26)	3 (39.73)	2 (18.07)	3 (5.61)	3 (1.50)	3 (5.44)
	2 (121.24)	3 (21.81)	2 (57.27)	3 (18.34)	2 (5.62)	2 (1.39)	2 (5.51)
	1 (122.26)	2 (21.88)	1 (57.41)	1 (18.77)	1 (5.98)	1 (1.63)	1 (5.54)

Table 9: Ranking of models, from lowest to highest squared sum of errors, for the periods indicated in the table.

Out-of-sample estimation for 2016 using data from 2015.

Model 1	Q1	Q2	Q3	Q4	$\sum u^2$	**Model 4**	Q1	Q2	Q3	Q4	$\sum u^2$	**Model 7**	Q1	Q2	Q3	Q4	$\sum u^2$
c2	0.3095	0.5047	0.1808	0.1637	1.1587	c2	0.1918	0.6008	0.0640	0.1681	1.0247	c2	0.1839	0.5080	0.1647	0.1729	1.0295
c3	0.2628	0.2761	0.1503	0.2105	0.8997	c3	0.2302	0.3643	0.0838	0.3209	0.9992	c3	0.1946	0.2803	0.1494	0.2069	0.8312
c4	0.1139	0.0419	0.0364	0.1088	0.3010	c4	0.1478	0.1305	0.0554	0.1822	0.5159	c4	0.1028	0.0464	0.0373	0.1125	0.299
c5	0.1471	0.0270	0.0356	0.0870	0.2967	c5	0.2178	0.0581	0.0629	0.1792	0.5180	c5	0.1580	0.0356	0.0354	0.0895	0.3185
c6	0.1022	0.0865	0.0750	0.1248	0.3885	c6	0.1487	0.0222	0.0881	0.2023	0.4613	c6	0.0999	0.0807	0.0711	0.1199	0.3716
$\sum u^2$	0.9355	0.9362	0.4781	0.6948	3.0446	$\sum u^2$	0.9363	1.1759	0.3542	1.0526	3.519	$\sum u^2$	0.7392	0.9511	0.4579	0.7018	2.85

Model 2	Q1	Q2	Q3	Q4	$\sum u^2$	**Model 5**	Q1	Q2	Q3	Q4	$\sum u^2$	**Model 8**	Q1	Q2	Q3	Q4	$\sum u^2$
c2	0.4351	0.7105	0.1991	0.2249	1.5696	c2	0.1803	0.4125	0.0815	0.1878	0.8621	c2	0.3138	0.7404	0.2277	0.2080	1.4899
c3	0.3443	0.4176	0.1731	0.2020	1.1370	c3	0.2201	0.1190	0.1091	0.3981	0.8463	c3	0.2699	0.4543	0.1377	0.1559	1.0178
c4	0.1600	0.0867	0.0483	0.1101	0.4051	c4	0.1449	0.0557	0.0733	0.1869	0.4608	c4	0.1396	0.1128	0.0222	0.0986	0.3732
c5	0.1640	0.0256	0.0327	0.0789	0.3012	c5	0.2077	0.0412	0.0831	0.1829	0.5149	c5	0.1796	0.0404	0.0562	0.1137	0.3899
c6	0.0945	0.0907	0.1006	0.1857	0.4715	c6	0.1443	0.0113	0.0984	0.2069	0.4609	c6	0.1009	0.0247	0.0195	0.1157	0.2608
$\sum u^2$	1.1980	1.3311	0.5538	0.8016	3.8845	$\sum u^2$	0.8974	0.6397	0.4455	1.1626	3.1452	$\sum u^2$	1.0038	1.3726	0.4632	0.6918	3.5314

Model 3	Q1	Q2	Q3	Q4	$\sum u^2$	**Model 6**	Q1	Q2	Q3	Q4	$\sum u^2$	**Model 9**	Q1	Q2	Q3	Q4	$\sum u^2$
c2	0.1775	0.5466	0.0959	0.2135	1.0355	c2	0.2114	0.2513	0.2351	0.1617	0.8595	c2	0.74298	0.7263	0.0564	0.0954	1.3079
c3	0.2166	0.3964	0.1334	0.3707	1.1171	c3	0.2583	0.1432	0.3787	0.3307	1.1109	c3	0.3256	0.4842	0.0687	0.1986	1.0771
c4	0.1309	0.1586	0.0810	0.2630	0.6341	c4	0.1799	0.1366	0.0308	0.1689	0.5162	c4	0.2249	0.1692	0.0371	0.0944	0.5256
c5	0.1873	0.0789	0.0796	0.3116	0.6574	c5	0.2603	0.0674	0.0409	0.1966	0.5652	c5	0.2059	0.0417	0.0444	0.1355	0.4275
c6	0.1292	0.0290	0.0802	0.3477	0.5861	c6	0.1741	0.0297	0.0560	0.1797	0.4395	c6	0.1050	0.0140	0.0387	0.1065	0.2642
$\sum u^2$	0.8414	1.2095	0.4702	1.5085	4.0296	$\sum u^2$	1.0840	0.6282	0.7415	1.0377	3.4914	$\sum u^2$	1.2912	1.4355	0.2453	0.6304	3.6024

Table 10: Squared sum of errors of the out-of-sample estimations for each model for 2016 using data from 2015. Red numbers are the lowest squared errors.

Out-of-sample estimation for 2016 using data from 2014 and 2015.

Model 1	Q1	Q2	Q3	Q4	$\sum u^2$
c2	0.3369	1.1168	0.5984	0.3756	2.4277
c3	0.2604	0.6553	0.4397	0.2749	1.6303
c4	0.0729	0.3036	0.1923	0.1255	0.6943
c5	0.0636	0.2347	0.1625	0.1003	0.5611
c6	0.0638	0.2496	0.2379	0.2216	0.7729
$\sum u^2$	0.7976	2.5600	1.6309	1.0979	6.0864

Model 2	Q1	Q2	Q3	Q4	$\sum u^2$
c2	0.5661	1.2537	0.5574	0.3953	2.7825
c3	0.4180	0.7388	0.4257	0.2226	1.8551
c4	0.1430	0.3274	0.1931	0.1305	0.7940
c5	0.0850	0.2336	0.1593	0.1022	0.5801
c6	0.0662	0.2547	0.2457	0.2900	0.8566
$\sum u^2$	1.2884	2.8082	1.5812	1.1906	6.8684

Model 3	Q1	Q2	Q3	Q4	$\sum u^2$
c2	0.2007	0.7673	0.2147	0.2496	1.4323
c3	0.1958	0.5393	0.2394	0.3600	1.3345
c4	0.0777	0.3424	0.1887	0.2511	0.8599
c5	0.1064	0.3052	0.1934	0.2395	0.8445
c6	0.0570	0.2476	0.1903	0.2467	0.7416
$\sum u^2$	0.6376	2.2017	1.0265	1.3469	5.2127

Model 4	Q1	Q2	Q3	Q4	$\sum u^2$
c2	0.1609	0.8292	0.2112	0.2532	1.4545
c3	0.1769	0.5729	0.2286	0.3537	1.3321
c4	0.0749	0.3539	0.1879	0.2519	0.8686
c5	0.1169	0.3048	0.1921	0.2384	0.8522
c6	0.0622	0.2483	0.1919	0.2452	0.7476
$\sum u^2$	0.5919	2.3091	1.0111	1.3424	5.2545

Model 5	Q1	Q2	Q3	Q4	$\sum u^2$
c2	0.0936	0.7738	0.2254	0.2889	1.3817
c3	0.1511	0.5703	0.2289	0.4654	1.4157
c4	0.0711	0.4693	0.2021	0.2244	0.9660
c5	0.1115	0.4314	0.2287	0.2095	0.9811
c6	0.0670	0.3382	0.2055	0.2271	0.8378
$\sum u^2$	0.4944	2.5830	1.0897	1.4154	5.5825

Model 6	Q1	Q2	Q3	Q4	$\sum u^2$
c2	0.2242	1.2140	0.4545	0.1872	2.0799
c3	0.1774	0.7588	0.4973	0.2751	1.7086
c4	0.0674	0.5031	0.1363	0.1984	0.9052
c5	0.1147	0.3650	0.1638	0.1872	0.8307
c6	0.0510	0.3181	0.1333	0.1985	0.7009
$\sum u^2$	0.6348	3.1591	1.3852	1.0465	6.2256

Model 7	Q1	Q2	Q3	Q4	$\sum u^2$
c2	0.4077	1.0941	0.4959	0.3470	2.3447
c3	0.2778	0.6529	0.4795	0.2643	1.6745
c4	0.0647	0.3154	0.1940	0.1316	0.7057
c5	0.08550	0.2285	0.1720	0.1027	0.5882
c6	0.0485	0.2314	0.2913	0.2316	0.8028
$\sum u^2$	0.8837	2.5222	1.6327	1.0772	6.1158

Model 8	Q1	Q2	Q3	Q4	$\sum u^2$
c2	0.3895	1.4258	0.6547	0.8427	3.3127
c3	0.3561	0.8740	0.5166	0.4591	2.2058
c4	0.0614	0.2984	0.1788	0.0679	0.6065
c5	0.1222	0.2990	0.1397	0.1367	0.6976
c6	0.0490	0.1649	0.2083	0.0681	0.4903
$\sum u^2$	0.9781	3.0620	1.6980	1.5745	7.3126

Model 9	Q1	Q2	Q3	Q4	$\sum u^2$
c2	0.4089	1.4193	0.4629	0.3802	2.6713
c3	0.2343	0.9294	0.3524	0.2240	1.7401
c4	0.1817	0.3486	0.2109	0.1001	0.8413
c5	0.1734	0.2254	0.1425	0.0894	0.6307
c6	0.0492	0.1747	0.1733	0.1016	0.4988
$\sum u^2$	1.0475	3.0974	1.3421	0.8953	6.3823

Table 11: Squared sum of errors of the out-of-sample estimations for each model for 2016 using data from 2014 and 2015. Red numbers are the lowest squared errors.

Out-of-sample estimation for 2017 using data from 2016.

Model 1	Q1	Q2	Q3	Q4	$\sum u^2$
c2	0.3018	0.0304	0.0640	0.2668	0.6630
c3	0.2993	0.0302	0.0634	0.2663	0.6592
c4	0.3006	0.0308	0.0637	0.2689	0.6640
c5	0.3029	0.0309	0.0662	0.2687	0.6687
c6	0.3041	0.0311	0.0673	0.2685	0.6710
$\sum u^2$	1.5087	0.1535	0.3246	1.3392	3.3260

Model 4	Q1	Q2	Q3	Q4	$\sum u^2$
c2	0.3009	0.0304	0..0673	0.1063	0.5049
c3	0.3004	0.0303	0.0648	0.1084	0.5039
c4	0.2993	0.0306	0.0645	0.1126	0.5070
c5	0.2987	0.0310	0.0633	0.1078	0.5008
c6	0.3199	0.0312	0.0648	0.1116	0.5275
$\sum u^2$	1.5193	0.1535	0.3246	0.5468	2.5442

Model 7	Q1	Q2	Q3	Q4	$\sum u^2$
c2	0.3012	0.0304	0.0594	0.2594	0.6506
c3	0.2982	0.0304	0.0646	0.2645	0.6571
c4	0.3044	0.0307	0.0668	0.2693	0.6712
c5	0.3038	0.0310	0.0710	0.2684	0.6742
c6	0.3060	0.0310	0.0702	0.2655	0.6727
$\sum u^2$	1.5137	0.1536	0.3315	1.3273	3.3261

Model 2	Q1	Q2	Q3	Q4	$\sum u^2$
c2	0.3019	0.0302	0.0638	0.2682	0.6641
c3	0.2994	0.0301	0.0634	0.2670	0.6599
c4	0.3005	0.0308	0.0638	0.2688	0.6639
c5	0.3028	0.0311	0.0664	0.2682	0.6685
c6	0.3040	0.0313	0.0675	0.2680	0.6708
$\sum u^2$	1.5086	0.1535	0.3249	1.3402	3.3272

Model 5	Q1	Q2	Q3	Q4	$\sum u^2$
c2	0.3032	0.0314	0.0642	0.2387	0.6375
c3	0.2935	0.0334	0.0652	0.3044	0.6965
c4	0.2997	0.0324	0.0614	0.2488	0.6423
c5	0.3005	0.0324	0.0667	0.2485	0.6481
c6	0.3226	0.0335	0.0787	0.2619	0.6867
$\sum u^2$	1.5195	0.1631	0.3362	1.2924	3.3112

Model 8	Q1	Q2	Q3	Q4	$\sum u^2$
c2	0.3018	0.0303	0.0573	0.2661	0.6555
c3	0.2966	0.0374	0.0592	0.2659	0.6591
c4	0.3032	0.0306	0.0612	0.2729	0.6679
c5	0.3122	0.0309	0.0908	0.2800	0.7139
c6	0.3115	0.0314	0.1148	0.2712	0.7294
$\sum u^2$	1.5253	0.1611	0.3833	1.3561	3.4258

Model 3	Q1	Q2	Q3	Q4	$\sum u^2$
c2	0.2958	0.0304	0.0577	0.2679	0.6518
c3	0.2950	0.0302	0.0638	0.2673	0.6563
c4	0.2979	0.0308	0.0718	0.2696	0.6701
c5	0.3021	0.0309	0.0882	0.2691	0.6903
c6	0.3048	0.0311	0.1048	0.2685	0.7092
$\sum u^2$	1.4956	0.1534	0.3862	1.3423	3.3775

Model 6	Q1	Q2	Q3	Q4	$\sum u^2$
c2	0.3064	0.0321	0.0639	0.2653	0.6677
c3	0.3217	0.0340	0.0564	0.2664	0.6785
c4	0.3050	0.0320	0.0575	0.2719	0.6664
c5	0.3078	0.0317	0.0906	0.2732	0.7033
c6	0.3085	0.0366	0.0870	0.2737	0.7058
$\sum u^2$	1.5495	0.1663	0.3555	1.3505	3.4218

Model 9	Q1	Q2	Q3	Q4	$\sum u^2$
c2	0.3031	0.0312	0.0630	0.1934	0.5907
c3	0.2995	0.0377	0.0657	0.1918	0.5947
c4	0.3004	0.0307	0.0635	0.1977	0.5923
c5	0.2966	0.0351	0.0725	0.1987	0.6029
c6	0.3144	0.0469	0.0639	0.1993	0.6245
$\sum u^2$	1.5139	0.1816	0.3286	0.9810	3.0051

Table 12: Squared sum of errors of the out-of-sample estimation for each model for 2017 using data from 2016. Red numbers are the lowest squared errors.

Out-of-sample estimation for 2017 using data from 2015 and 2016.

Model 1	Q1	Q2	Q3	Q4	$\sum u^2$	**Model 4**	Q1	Q2	Q3	Q4	$\sum u^2$	**Model 7**	Q1	Q2	Q3	Q4	$\sum u^2$
C2	0.1996	0.1612	0.1791	0.0605	0.6004	c2	0.1700	0.1502	0.1832	0.0610	0.5644	c2	0.2042	0.1589	0.1818	0.0609	0.6058
c3	0.2025	0.1622	0.1799	0.0613	0.6059	c3	0.1605	0.1476	0.1838	0.0612	0.5531	c3	0.2034	0.1645	0.1820	0.0618	0.6117
c4	0.2085	0.1638	0.1796	0.0610	0.6129	c4	0.1599	0.1464	0.1842	0.0605	0.551	c4	0.2070	0.1612	0.1807	0.0611	0.6100
c5	0.2172	0.1660	0.1797	0.0610	0.6239	c5	0.1591	0.1469	0.1846	0.0607	0.5513	c5	0.2140	0.1622	0.1795	0.0628	0.6185
c6	0.2232	0.1694	0.1773	0.0616	0.6315	c6	0.1612	0.1481	0.1830	0.0621	0.5544	c6	0.2288	0.1642	0.1761	0.0608	0.6299
$\sum u^2$	1.0510	0.8226	0.8956	0.3054	3.074	$\sum u^2$	0.8106	0.7393	0.9188	0.3055	2.7742	$\sum u^2$	1.0574	0.8111	0.9001	0.3074	3.076

Model 2	Q1	Q2	Q3	Q4	$\sum u^2$	**Model 5**	Q1	Q2	Q3	Q4	$\sum u^2$	**Model 8**	Q1	Q2	Q3	Q4	$\sum u^2$
c2	0.2157	0.1574	0.1805	0.0607	0.6148	c2	0.1623	0.1465	0.1797	0.0571	0.5456	c2	0.2165	0.1629	0.1817	0.0609	0.6220
c3	0.2046	0.1596	0.1807	0.0608	0.6057	c3	0.1500	0.1511	0.1803	0.0576	0.5390	c3	0.2054	0.1649	0.1818	0.0609	0.6130
c4	0.2047	0.1634	0.1797	0.0605	0.6083	c4	0.1491	0.1443	0.1793	0.0573	0.5300	c4	0.2165	0.1680	0.1811	0.0604	0.6260
c5	0.2270	0.1682	0.1787	0.0608	0.6347	c5	0.1601	0.1435	0.1778	0.0608	0.5422	c5	0.2158	0.1677	0.1807	0.0616	0.6258
c6	0.2543	0.1740	0.1764	0.0611	0.6658	c6	0.1831	0.1501	0.1772	0.0604	0.5708	c6	0.2328	0.1728	0.1779	0.0625	0.6460
$\sum u^2$	1.1063	0.8231	0.8960	0.3038	3.1292	$\sum u^2$	0.8046	0.7355	0.8942	0.2932	2.7275	$\sum u^2$	1.0869	0.8363	0.9031	0.3063	3.1326

Model 3	Q1	Q2	Q3	Q4	$\sum u^2$	**Model 6**	Q1	Q2	Q3	Q4	$\sum u^2$	**Model 9**	Q1	Q2	Q3	Q4	$\sum u^2$
c2	0.1583	0.1435	0.1778	0.0613	0.5409	c2	0.1942	0.1359	0.1771	0.0607	0.5679	c2	0.2001	0.1636	0.1802	0.0622	0.6061
c3	0.1492	0.1425	0.1776	0.0609	0.5302	c3	0.1924	0.1441	0.1799	0.0607	0.5771	c3	0.2006	0.1640	0.1816	0.0603	0.6065
c4	0.1462	0.1435	0.1766	0.0604	0.5267	c4	0.1917	0.1368	0.1757	0.0599	0.5641	c4	0.2100	0.1660	0.1809	0.0598	0.6167
c5	0.1637	0.1457	0.1755	0.0605	0.5454	c5	0.1952	0.1442	0.1748	0.0614	0.5756	c5	0.2079	0.1754	0.1802	0.0612	0.6247
c6	0.2002	0.1505	0.1728	0.0608	0.5843	c6	0.2802	0.1431	0.1733	0.0628	0.6594	c6	0.1994	0.1750	0.1776	0.0607	0.6127
$\sum u^2$	0.8178	0.7256	0.8803	0.3039	2.7276	$\sum u^2$	1.0537	0.7041	0.8807	0.3054	2.9439	$\sum u^2$	1.0180	0.8439	0.9005	0.3042	3.0666

Table 13: Squared sum of errors of the out-of-sample estimation for each model for 2017 using data from 2015 and 2016. Red numbers are the lowest squared errors.

2-factor model.								
Estimation of parameters								
α	κ	σ	$2\pi\omega$	correlation	Ax	Ay	Ax2	Ay2
3.2262	0.0018	0.0000	$2\pi 0.1316$	-0.0776	0.9728	0.9623	1.0910	1.0146
$\sum u^2$	C2	C3	C4	C5	C6	TOTAL		
	0.0009	11.0571	0.0034	0.0056	0.0078	11.0748		

Table 14: Estimation of the parameters of the two-factor model and squared errors for each futures. The sample period is from 2013 until 2018.

Appendix of Figures

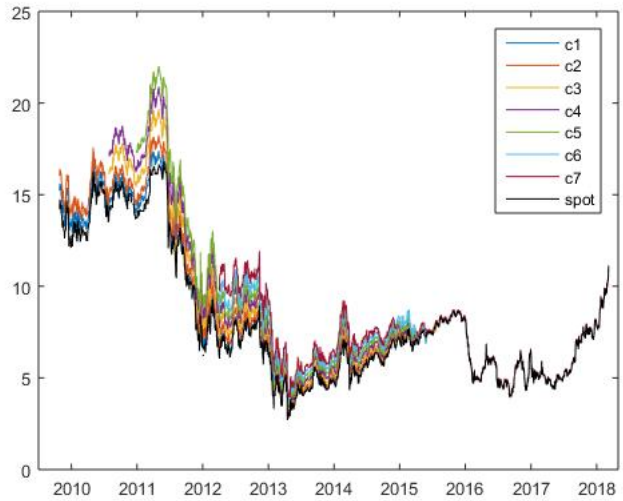

Figure 1: Time series of prices of futures c1 to c7 and spot price for the whole sample (first available data until 2018).

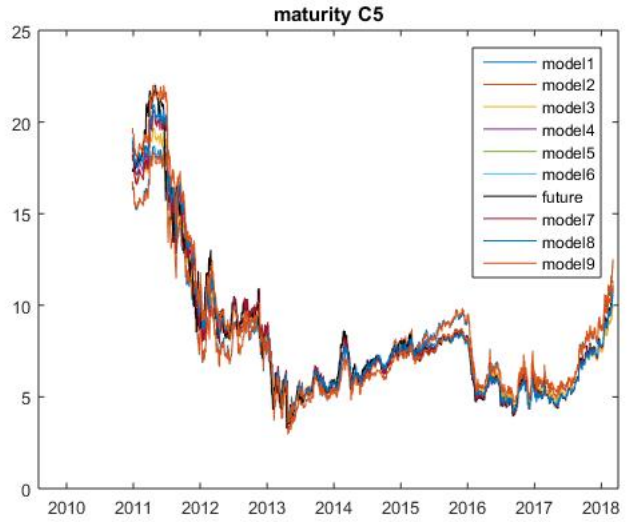

Figure 2: Spot price and fitted prices for the futures c5 under the nine models. The sample period for this futures is from 29, December, 2010 until 9, March, 2018.

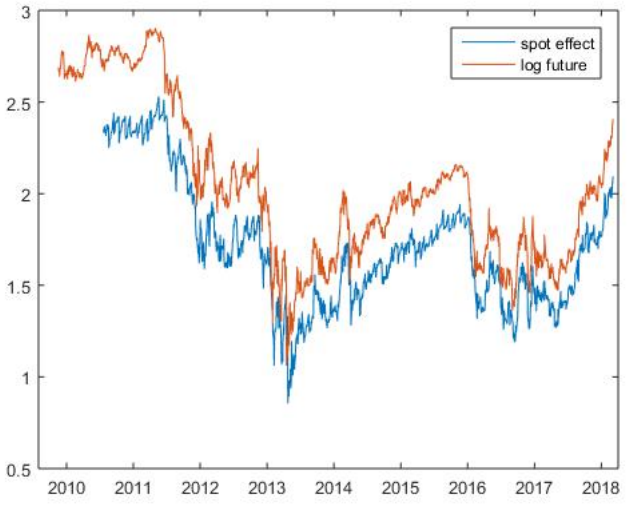

Figure 3: Spot effect and observed log-price of the futures c2. The sample period is from 27, October, 2009 until 9, March, 2018.

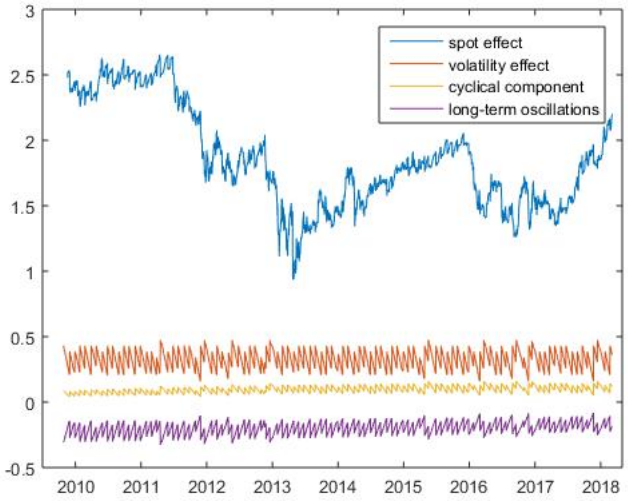

Figure 4: Decomposition of the four effects in model 5 for the futures c2. The sample period is from 27, October, 2009 until 9, March, 2018.

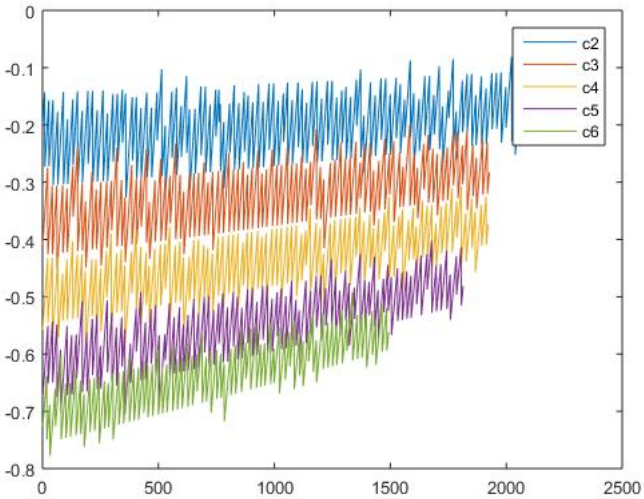

Figure 5: Estimation of the level of mean-reversion (long-term swing) for the prices of the futures c2 to c6.

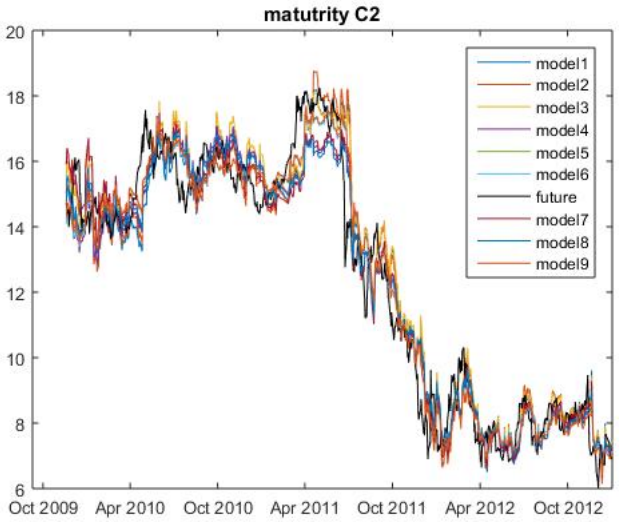

Figure 6: Spot price and fitted prices for the futures c2 under the nine models. The sample period for this futures is from 27, October, 2009 until 9, March, 2018.

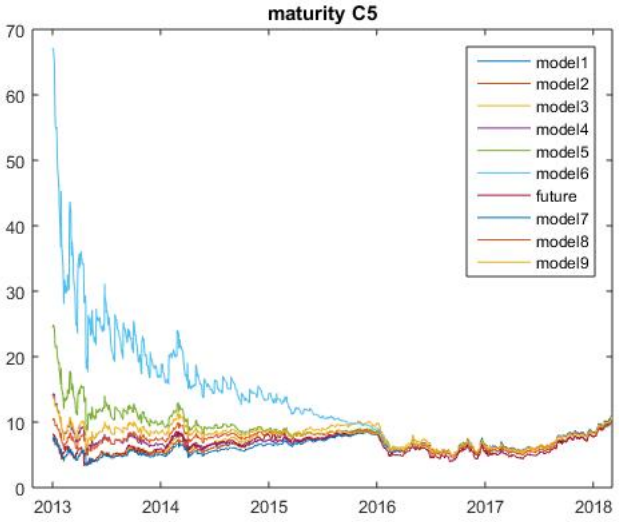

Figure 7: Spot price and fitted prices for the futures c5 under the nine models. The sample period for this futures is from 2013 until 9, March, 2018.

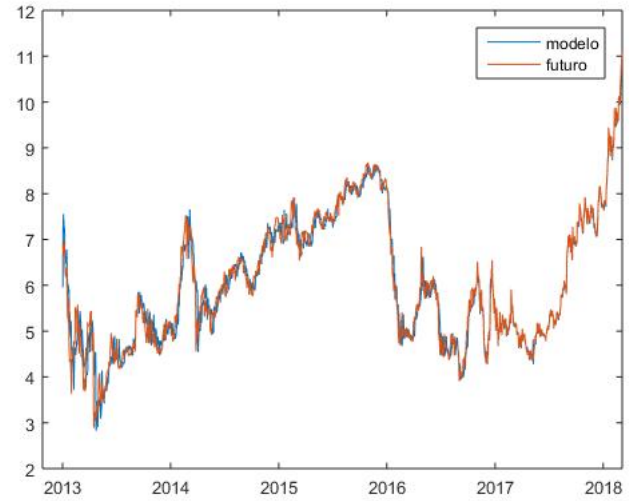

Figure 8: Spot price and fitted price for the futures c5 under the two-factor model. The sample period for this futures is from 2013 until 9, March, 2018.

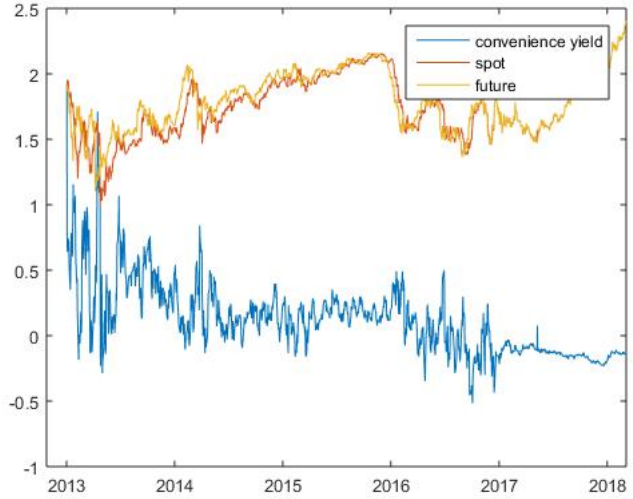

Figure 9: Price of the futures c5, spot price, and convenience yield from 2013 until 9, March, 2018.