Better to grow or better to improve? Measuring environmental efficiency in OECD countries with a Stochastic Environmental Kuznets Frontier

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ABSTRACT

The standard approach to the Environmental Kuznets Curve (EKC) holds that as a country develops and GDP per capita grows environmental degradation initially increases but eventually it reaches a turning point where environmental degradation begins to decline. Environmental degradation takes many forms, one of them being emissions of harmful gases. According to the EKC concept, a country can reduce emissions by 'growing'. The standard approach implicitly assumes that a country emits as little as possible for its economic development, whereas in reality, a country might emit above the best attainable level of emissions. Therefore, emissions could be reduced before and after the turning point by becoming more environmentally efficient – i.e., 'improving' the emissions level. This article proposes a Stochastic Environmental Kuznets Frontier (SEKF) which is estimated for CO_2 emissions for OECD countries and used to benchmark each country before and after the turning point differently, thus, indicating how a country could 'grow' and/or 'improve' to reduce its CO_2 emissions. Additionally, we analyse the role of the stringency of environmental policies in reducing a country's carbon inefficiency measured by the distance from the benchmark EKC and find widespread carbon inefficiencies that could be reduced by more stringent market-based environmental policies.

JEL codes: O44, Q56, Q54, C13, C33

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1. Introduction

The debate on the relationship between economic development and environmental quality dates back more than fifty years. In the early phases of the debate the prevailing view was that economic growth is a threat to the environment. This position was echoed by the famous book "The Limits to Growth" (Meadows et al., 1972): higher levels of economic activity imply increased extraction of natural resources, accumulation of waste, concentration of pollutants that would exceed the carrying capacity of the biosphere and result in a degradation of environmental quality and a decline in human welfare, despite rising incomes. To save the environment and even economic activity from itself, economic growth must cease and the world must make a transition to a steady-state economy (Daly, 1991).

This was a difficult position for industrializing countries; hence, the Kyoto Protocol resulted in developing countries making no commitment to reduce their greenhouse gas emissions on the grounds that the industrialization process should have no constraints, especially on energy production and consumption. The position was also difficult for developed countries which championed the welfare-increasing goal of economic growth (bringing with it poverty reduction, improved health conditions, among other benefits), over the reduction of environmental degradation. This lasted until the actual damage to the environment produced by various pollutants – especially local ones – or the increasing perception of the damage – as in the case of greenhouse gas emissions – become too evident and thus prompted governments to act.

In fact, contrary to the Malthusian view that environmental limitations are significant enough to prevent sustained growth in consumption and production, there are those who believe that environmental factors and resource constraints pose no limitation to economic growth. According to this view, the fastest road to environmental improvement is along the path of economic growth: higher incomes increase the demand for less material intensive goods and services and at the same time bring about an increased demand for environmental protection measures. Famous in this respect is the quotation from Beckerman (1992): ''The strong correlation between incomes and the extent to which environmental protection measures are adopted demonstrate that in the longer run the surest way to improve your environment is to become rich'' (p. 495).

A milder position holds that environmental limitations will exert a "drag" on economic growth. This environmental drag is caused by natural resource limitations and the various negative

effects of pollution on productivity and human well-being (Hepburn and Bowen, 2012). According to its proponent (Nordhaus, 1992), the environmental drag is the difference between national income growth when resources are superabundant (but not free) and there is no pollution, and actual national income growth with scarce resources and pollution.

The synthesis between these different positions came about at the beginning of the 1990s when several researchers collected rich datasets on emissions and concentrations of several pollutants and on measures of sustainability which for the first time enabled the econometric investigation of the relationship between growth and environment. To accommodate the view of both pessimists and optimists, a non-linear relationship between environmental degradation and economic activity was fitted to the data and became known as the Environmental Kuznets Curve (EKC) hypothesis, being analogous to the historical relationship between income distribution and income growth initially proposed by Kuznets (1955). A bell-shaped (or inverted U-shaped) curve implies that, starting from low-income levels, environmental degradation tends to increase but at a slower pace. After a certain level of income (which typically differs across pollutants) – the 'turning point' – environmental degradation starts to decline as income further increases. Again, in the words of Beckerman (1992), ''there is clear evidence that, although economic growth usually leads to environmental degradation in the early stages of the process, in the end the best – and probably the only – way to attain a decent environment in most countries is to become rich'' (p. 496). One explanation is that generally, economic growth at least partly accounts for technological and intellectual advances, which prompts an increased demand for environmental protection due to the presumed luxury good nature of the environment itself, and brings about structural changes in the composition of production and consumption activities toward less material- and energy-intensive ones.

Environmental degradation takes many forms, a relevant one being emissions of harmful gases. An inverted U-shaped EKC suggests that as a country develops and GDP per capita grows there is an initial increase in emissions but eventually it will reach a point where emissions will begin to decline – thus the main way for a country to reduce emissions is to continue to '*grow*'. However, this implicitly assumes that the country is on the EKC (similar to the way standard introductory economics textbooks assume that a firm is always on a cost curve) whereas in reality this might not be the case and a country, for various reasons, might be emissions inefficient and above the best attainable EKC (similar to a firm being inefficient if it is actually above its cost curve). In this case emissions could be reduced before and after the turning point by becoming more emissions efficient – i.e., to '*improve*'. We therefore propose, in this paper,

an approach that estimates an Environmental Kuznets Frontier (EKF) to represent the 'best' EKC across a number of OECD countries in order to benchmark each country against. Thus, giving an indication of how a country could '*grow*' and/or '*improve*' to reduce emissions.

To achieve this, we introduce the concept of a Stochastic Environmental Kuznets Frontier (SEKF) and develop a framework that allows us to empirically analyse both ways to reduce a country's emissions, that is via economic growth or through an improvement in emissions efficiency. This builds upon two strands of literature from 'environmental economics', and 'productivity and efficiency economics': the EKC and Stochastic Frontier Analysis (SFA), respectively. As we demonstrate, the SEKF framework allows us to estimate an inverted U-shaped Environmental Kuznets Frontier (EKF) that represents the beast feasible path for a country to '*grow*' in order to reduce emissions, which is also used as a benchmark to measure a country's environmental inefficiency showing the shortest distance from the EKF indicating the way a country could '*improve*' in order to reduce the level of emissions.

Furthermore, we build upon a further strand of literature from 'environmental economics' by analysing the role and the stringency of environmental policies in reducing a country's emissions inefficiency measured by the distance from the benchmark EKF. Such emission reductions brought about by the re-organization of production and distribution within and outside the firm, changes in the energy mix, energy conservation and behavioural changes toward energy savings are all cases where in principle it is possible to become more efficient at unchanged GDP. All these changes are likely to be policy-induced, which we explore via the introduction of environmental policy stringency measure as a driver of countries' emissions inefficiency. Thus, the conceptual approach introduced as well as the empirical results found in this paper contribute to the academic literature but will also be of interest to policy makers given the analysis of the dilemma whether to '*grow*' or to '*improve*'.

Based on this new approach, we present an empirical application that estimates a SEKF with CO_2 as emissions using a cross-country analysis for the relatively homogenous group represented by OECD countries. The results support to the idea of a benchmark EKF that is inverted-U shaped with a reasonable estimate of the turning point of per capita GDP. We then assess whether the distance of a country at a point in time from the efficiency frontier is, or could be, affected by environmental policy. To that end, we assume that the variance of the stochastic inefficiency term depends on an indicator of environmental policy stringency using a well-known index provided by the OECD which is comprised of both market-based and non-market-based policy instruments. We find support for a significantly negative impact of

environmental policy stringency on the degree of carbon inefficiency but it is limited to market-based policy instruments. The preferred model shows how climate policies such as carbon pricing measures, subsidies to clean energy sources and the like are potentially capable of reducing the distance of a country from the efficiency frontier. We find that, when the environmental policy indicator goes up by 1 unit (the index ranges from 0 to 6), there is, on average, a reduction in CO_2 emissions of 20%, with strongest average impacts for Ireland, Finland, Belgium, and Norway and weakest impacts for Spain, Germany, France, Denmark, and Portugal. However, environmental policy to curb CO_2 emissions has become more stringent over time. Indeed, when we split the sample between the first decade 1990-2000 and the second one 2001-2012, we find that in every country policy action becomes stronger. Indeed, for nearly all countries the impact becomes stronger in the second period relative to the first one.

The paper is organized as follows. Section 2 presents a brief review of the relevant literature. The conceptual approach and the econometric methodology are discussed in Section 3. Section 4 describes the data and Section 5 presents the empirical results. Section 6 contains some concluding remarks.

2. Selected literature review

As highlighted above, we develop the concept of a SEKF building on three different strands of literature. This section therefore reviews briefly the key aspects of each strand given that a thorough review when space is limited is impossible.

2.1 Environmental Kuznets Curve (EKC)

Much has been written on the growth–environment relationship and on the EKC. Since the spate of initial influential studies by Grossman and Krueger (1991, 1995), Shafik and Bandyopadhyay (1992), and Panayotou (1993, 1995), the literature has mushroomed making this probably the most empirically investigated theme in the field of environmental economics.

The environmental indicators that have been used in the EKC literature can be grouped as air quality, water quality and other environmental quality indicators (Galeotti, 2007). For the first category there is strong, but not overwhelming, empirical evidence in favour of an EKC. A distinction conventionally made in the literature is between local and global air pollutants. Indicators of urban and local air quality (sulphur dioxide, suspended particulate matters,

carbon monoxide and nitrous oxides) generally show an inverted U-shaped relationship with income. There are, however, major differences across indicators as to the turning point of the EKC and differences occur also for the same pollutant across alternative studies. When emissions of air pollutants have little direct impact on the population the literature generally finds mixed evidence. This holds especially for emissions of global pollutants such as carbon dioxide, which sometimes are found to monotonically increase with income or start declining at income levels well beyond the observed range (see e.g., Stern, 2017; Shahbaz and Sinha, 2019).

Carbon dioxide (CO_2) emissions play an important role in global warming as they represented around 72% of total greenhouse gas emissions in 2019 (Olivier and Peters, 2020). Burning fossil fuels to promote economic development continues to significantly contribute to CO_2 emissions, although several strategies have been put in place to reduce emissions, consistent with the Kyoto Protocol and Paris Agreement. Since the initial support for the EKC in the pioneering studies of Grossman and Krueger (1993) and Panayotou (1993), various studies have reached mixed conclusions regarding the existence of the EKC including papers focused on OECD countries (see e.g., Martinez-Zarzoso and Bengochea-Morancho, 2004; Galeotti et al., 2006; Cho et al., 2014; Bilgili et al., 2016; Alvarez-Herranz et al., 2017; Churchill et al., 2018).

2.2 Stochastic Frontier Analysis (SFA)

Like the EKC literature, estimating efficient frontiers has a long history using both linear programming methods such as Data Envelopment Analysis (DEA) (Charnes et al., 1978; Banker et al., 1984) and econometric methods such as SFA (Aigner et al., 1977; Meeusen and van den Broeck, 1977). Given the objectives of this research we focus on the SFA framework, which has been applied in several areas.

Filippini and Hunt (2011) estimate a panel frontier aggregate energy demand function for 29 OECD countries over the period 1978 to 2006 using parametric SFA. Unlike standard energy demand econometric estimation, the energy efficiency of each country is also modelled and it is argued that this represents a measure of the underlying efficiency for each country over time, as well as the relative efficiency across the OECD countries. Stern (2012) uses a stochastic production frontier to model energy efficiency trends in 85 countries over a 37-year period. Energy efficiency is measured using an energy distance function approach where the country using the least energy per unit output, given its mix of outputs and inputs, defines the global production frontier. A country's relative energy efficiency is given by its distance from the

frontier. Robaina-Alvesa et al. (2015) specify a new stochastic frontier model where GDP and greenhouse gas emissions are the outputs, while capital, labour, fossil fuels and renewable energy consumption are regarded as inputs. A new maximum entropy approach to assess technical efficiency, which combines information from DEA and the structure of composed error from the stochastic frontier approach without requiring distributional assumptions, is used.

Looking specifically at applying frontier analysis to environmental issues, Zaim and Taskin (2000) use a production frontier where real GDP is the desirable output and CO_2 emissions the only undesirable output of a technology using employment and capital stock as inputs. The environmental efficiency index obtained using non-parametric techniques aims at measuring the opportunity cost of adopting environmentally desirable technologies for OECD countries. Orea and Wall (2017) also use SFA to measure eco-efficiency for a sample of 50 Spanish dairy farmers. However, no previous study, as far as we are aware, has used SFA with emissions as the dependent variable as we do in this paper when estimating a SEKF.

2.3 Role and stringency of environmental policies

As argued in the introduction, the improvements in environmental performance indicators are likely to come from environmental policies. Hence, besides the work on the EKC hypothesis and SFA, this paper brings together a third area of the environmental economics literature, dealing with the role and stringency of environmental policies. In terms of role, previous studies have investigated the impact of environmental regulation on several key economic outcomes, such as productivity, competitiveness, and innovation of firms and sectors along the lines of the so-called Porter hypothesis (Porter, 1991; Porter and van der Linde, 1995; Jaffe and Palmer, 1997; Rubashkina et al., 2015). As for stringency, the main problem is to find appropriate empirical proxies for the commitment to, and stringency of, environmental policy (Brunel and Levinson, 2013; Galeotti et al., 2020). A composite indicator with wide coverage of policy instruments, time and countries is the OECD Environmental Policy Stringency (EPS) database (Botta and Koźluk, 2014), which has a wide coverage of policies and measures, as well as the availability for OECD countries; hence, this paper takes advantage of this indicator.

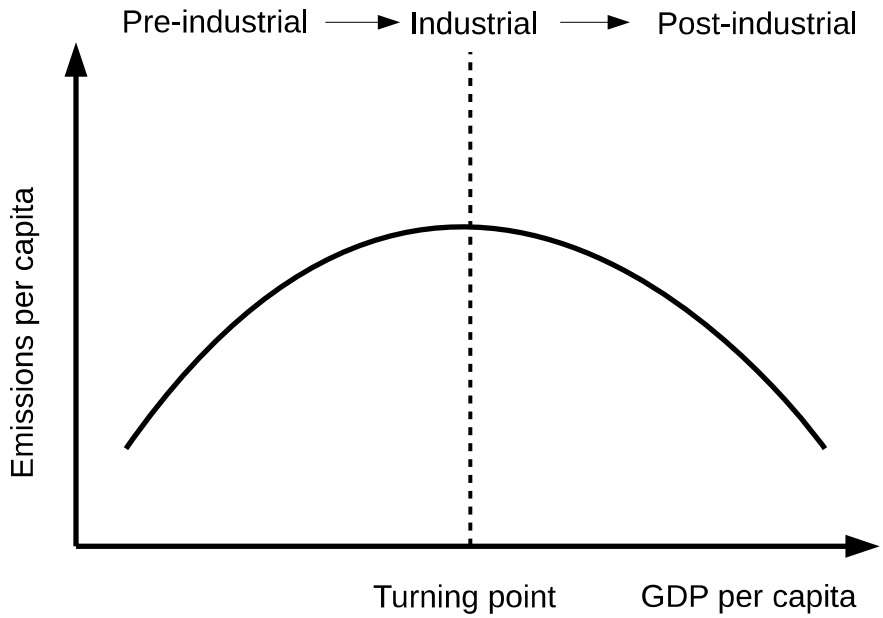
Our approach, therefore builds on this previous work by being, as far as we are aware, the first to explicitly link environmental policy stringency to carbon inefficiency which we estimate via our new SEKC framework The next section therefore introduces the details of the conceptual SEKF framework adopted in this research building on the three strands briefly discussed above.

3. Environmental Kuznets Frontier and Environmental efficiency

3.1 Conceptual formulation

Figure 1 illustrates the standard EKC hypothesis. Starting from low (per capita) income levels a country's (per capita) emissions will tend to increase but at a slower pace. After a certain level of income – the "turning point" – environmental quality starts to improve as (per capita) emissions decline with income increasing.[1] If the data refer to many countries for a period of time the EKC divides countries into different stages of economic development and environmental degradation. The post-industrial portion of the EKC is a very appealing concept in the sense that economies grow richer while reducing emissions (Figure 1).

Figure 1: Standard environmental Kuznets curve

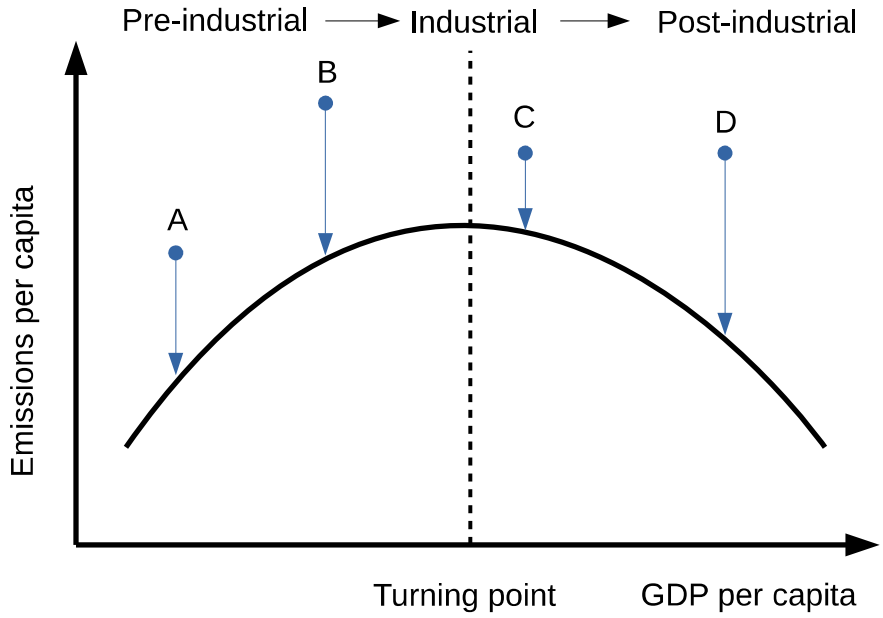


However, we argue that the EKF is a theoretical construct which is the lower bound of emissions given economic development. Thus, it is important to also take into account the ability of economies to reduce emissions by becoming more environmentally efficient. The solid curve in Figure 2 therefore illustrates the EKF or the theoretical minimum of emissions for a given level of economic development. The figure shows four hypothetical countries represented by points

[1] This section introduces the conceptual basis for introducing the SEKC and it should be noted that the approach could potentially be applied using any pollutant emissions or measure of environmental degradation. In the empirical application of this new procedure later in the paper we use CO_2 emissions as the measure of environmental degradation.

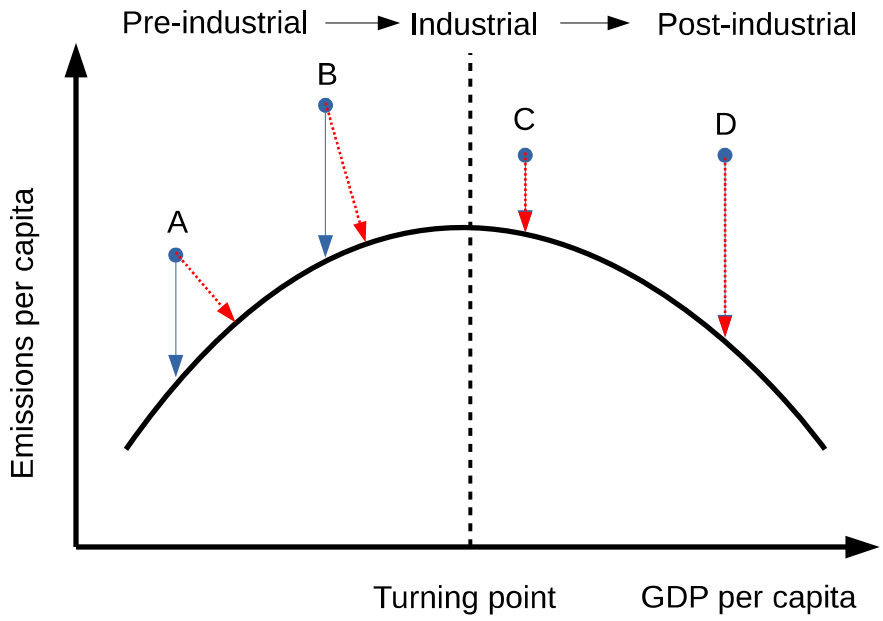
A to D in different stages of economic development. At a given level of economic development, their ability to reach the minimum possible level of emissions is given by the vertical distance from the observation to the solid curve. Country A is relatively closer to the EKF than country B. Country C is the closest to the possible minimum, while country D is quite far from the frontier and should be emitting much less for its level of economic development.

Figure 2: Environmental Kuznets frontier and the ability to reach the minimum possible level of emissions



It is however unreasonable to expect countries such as A and B to only strive to reduce emissions given the level of economic development. They are in the pre-industrial stage of development and naturally wish to expand further. It is therefore desirable to measure their ability to reduce emissions together with the ability to grow. Such correction can be made by measuring their ability to reach the minimum possible level of emissions not as a vertical, but as the shortest distance to the EKF. Figure 3 demonstrates this correction.

Figure 3: Environmental Kuznets frontier and the shortest distance to the minimum possible level of emissions



The approach taken here is that the vertical dotted (blue solid) line before (after) the turning point measures emissions inefficiency. The ability to reach the minimum possible level of emissions is unchanged for economies beyond the turning point and remains a vertical distance to the EKF from the observation. The relatively less developed economies whose economic development has not reached the turning point is measured by a non-vertical distance to the EKF. To reflect their determination to grow economically and to reduce emissions, their ability to reach the minimum possible level of emissions is measured by the shortest distance to the EKF.

3.2 Emissions Efficiency

Generally, we term the ability to limit environmental degradation for a given level of economic development as *environmental efficiency*. The difference to the previous literature that considered environmental efficiency is that we make it conditional on the level of economic development of a country. *Environmental inefficiency,* shown by the red dotted arrows in Figure 3, is measured by the shortest distance to the EKF for countries before the turning point and by the vertical distance to the EKF after the turning point. Thus, estimating a SEKF allows for the measurement of *emissions efficiency*.

3.3 Identification of Emissions Efficiency

To empirically analyse emissions efficiency, we need two components. First, we need to estimate the EKF and the turning point. Second, we need to identify emissions inefficiency. We show that this can be done in one step by augmenting the standard Stochastic Frontier (SF)

model. The SF approach posits the lower bound, which due to its stochastic nature still allows some observations to lie above the measured frontier. More specifically, the stochastic version of the EKF considered in the previous section can be written as:

$$E = F(Y; \boldsymbol{\beta}) + v + u, \tag{1}$$

where $F(.)$ is the functional form of the EKF determined by $\boldsymbol{\beta}$, the parameter vector to be estimated, E is emissions[2] per capita, and Y is GDP per capita. The observed level of E is higher than the minimum possible $F(Y; \boldsymbol{\beta})$, u is a positive term which measures the vertical distance to the EKF, and v is the usual error term which makes the frontier stochastic.

The term u in the specification (1) measures the vertical distance to the frontier $F(Y; \boldsymbol{\beta})$, which is shown as a blue arrow in Figure 3. Assuming that the frontier is a parabolic function, the turning point denoted by Y^T is obtained by solving $\partial E / \partial Y = 0$. The estimated turning point, therefore, depends on Y as well as $\boldsymbol{\beta}$, the estimation of which in turn will depend on how the distance to the frontier is measured. emissions inefficiency, denoted by u^*, is smaller than the vertical distance for countries represented by points such as A and B in Figure 3, that is, when $Y < Y^T$. Therefore, by assuming that emissions inefficiency (u^*) is the product of the vertical distance u and a "gap factor" denoted by h, which shows how low a country's GDP per capita is relative to the turning point Y^T, $u^* = u \times h$. Therefore:

$$h \text{ is } \begin{cases} < 1 & \text{for } Y < Y^T \\ = 1 & \text{otherwise.} \end{cases} \tag{2}$$

The gap factor, h, is multiplicative, the bigger is the gap between a country's GDP per capita and the turning point, the smaller is h. If a country's GDP per capita is at or to the right of the turning point, the gap factor h is equal to 1. To the left of the turning point, the bigger the gap the lower the h factor, which would be 1 if there is no gap.

The steps required to measure the emissions efficiency can be summarized as follows. First, we assume that there exists an EKF, which is a lower bound of emissions per capita for a given GDP per capita. The nature of the EKF is that it is upward sloping for the pre-industrial stage of

[2] As stated above a range of emissions could be considered such as CO_2, NO_X, SO_2, etc.

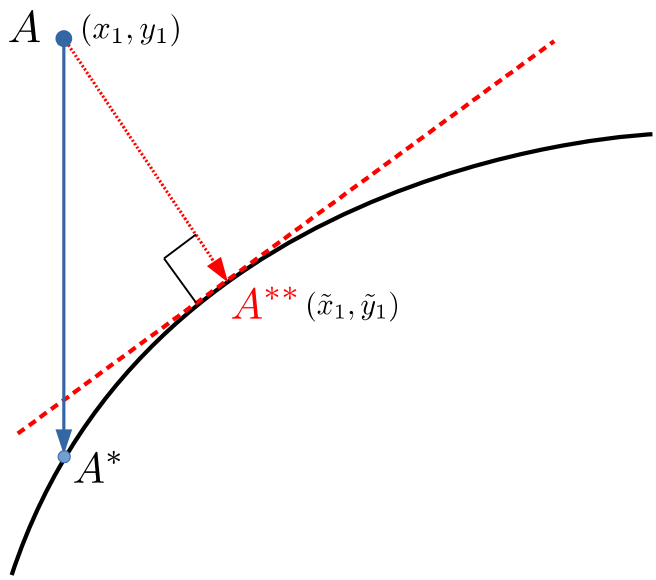
economic development and it is either downward sloping (or at the worst flat, see e.g., Galeotti, 2007) for the post-industrial stage. The transition from the pre- to post-industrial stage is the turning point.[3] This is achieved by assuming a parabolic EKF. Second, we posit that emissions inefficiency is a measure of how far away a country is from the EKF. Third, we postulate that the measurement will depend on a country's economic development. More specifically, if a country's economy can be considered to be post-industrial, we measure its ability to reduce emissions by the vertical distance to the EKF. If, on the other hand, a country is in a pre-industrial state, we measure its ability to reduce emissions by the closest distance to the EKF, which, due to the EKF being upward sloping for the pre-industrial stage, is shorter than the vertical distance. We call the factor by which the closest distance is shorter than the vertical distance the gap factor and denote it by h, which is discussed further in the next section.

3.4 The gap factor *h*

As highlighted above, the gap factor h will be closer to one for a pre-industrial economy that is closer to the turning point. In other words, the lower is the economic development of a country, the smaller is the distance factor, h. The next step therefore is to retrieve h in (2). Consistent with previous literature we assume that the EKF has a parabolic shape and therefore requires a framework to discover the closest distance to a parabola. Figure 4 focusses on the left-hand part of Figure 3.

[3] There is probably no abrupt turning point but rather a region, where the transition occurs. Below we estimate the confidence bounds of such a region.

Figure 4: Fragment of the Environmental Kuznets frontier and the exact solution to find *h*



The distance AA^* is the vertical distance, u. The shortest distance to a parabola $ax^2 + bx + c$ is shown by AA^{**}. If we know the coordinates of a point (x_1, y_1), then the (squared) distance to a point $(\tilde{x}_1, \tilde{y}_1)$ on the parabola is:

$$\begin{aligned} d^2 &= (\tilde{x}_1 - x_1)^2 + (\tilde{y}_1 - y_1)^2 \\ &= (\tilde{x}_1 - x_1)^2 + (a\tilde{x}_1^2 + b\tilde{x}_1 + c - y_1)^2 \end{aligned}$$

To find $\tilde{x}_1$, where the distance is the shortest, we set $\partial d^2 / \partial \tilde{x}_1 = 0$, which is a cubic equation with no analytical form. While the solution of the cubic equation is the exact solution for AA^{**}, it will be infeasible in estimation. In practice, we will consider an approximation.

Figure 5 shows the vertical distance u as in Figure 4 (the blue line) and the dotted tangent line to the parabola where $x = \tilde{x}_1$ (the red line). The dotted arrow is orthogonal to the dotted (red) tangent line. We approximate the dashed arrow distance AA^{**} by the dotted arrow distance AA^{***} (the green line). This approximation is good if the curvature of the EKF is not strong, as demonstrated for example in Figure 6, where A^{***} almost coincides with A^{**}. In this case h is expected to be close to 1.

Figure 5: Fragment of the Environmental Kuznets Frontier and the approximate solution to find *h*

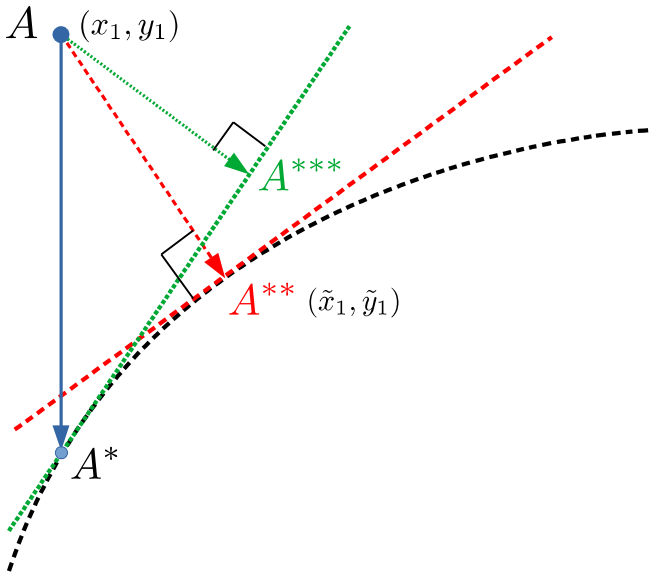
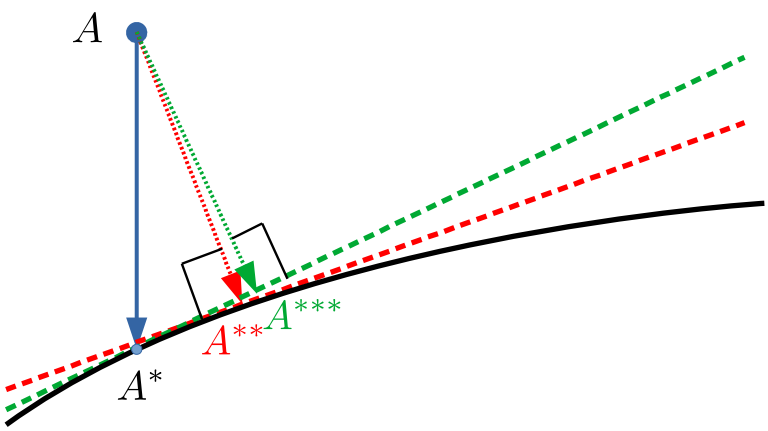


Figure 6: Fragment of the Environmental Kuznets Frontier with smaller curvature and the approximate solution to find *h*



For a parabolic EKF given by $ax^2 + bx + c$, the h in (2) can be approximated by $1/\left(\sqrt{1 + (b + 2ax_1)^2}\right)$.[4]

[4] Briefly, the shortest distance from a point (x_1, y_1) to the tangent line in point A^* is given by $AA^*/\sqrt{1 + (b + 2ax_1)^2}$. Since AA^* is equal to u, then $h = 1/\sqrt{1 + (b + 2ax_1)^2}$.

3.5 Stochastic Environmental Kuznets Frontier

This section introduces the SEKF which accounts for the possibility that $h < 1$ for countries that have not reached the turning point. We first present the model which extends the standard second-generation stochastic frontier model with two time-varying components. Then we consider the third- and fourth-generation stochastic frontier models, which take heterogeneity into account.[5]

Denoting per capita emissions with $e = E/P$, the second-generation stochastic frontier model can be generally written as:

$$lne_{it} = f(\cdot) + v_{it} + u_{it} \tag{3}$$

where country $i = 1, \dots, N$ is observed T_i times, so that the total number of observations is $\sum_{i=1}^{N} T_i$ (unbalanced panel). Model (3) is operationalised by taking logs of per capita emissions and real per capita GDP as a proxy for the level of economic development denoted as $y = GDP/P$. In addition, we follow the bulk of the EKC literature by parametrizing $f(\cdot)$ as a quadratic relationship, so that:

$$f(\cdot) = \beta_0 + \beta_1 lny_{it} + \beta_2 (lny)_{it}^2 + \mathbf{x}_{it}\boldsymbol{\gamma} \tag{4}$$

Note that the turning point is given by $e^{-\beta_1/(2\beta_2)}$.[6] Finally, in many cases, the EKC relationship includes controls other than GDP, denoted by the vector of variables $\mathbf{x}_{it}$.[7]

Following the earlier exposition, the emissions inefficiency is the product of the vertical distance u_i and the gap factor, which is time-varying and country-specific, $u_{it} = h_{it}u_i > 0$. It follows from the previous section that the gap factor is defined as follows:

[5] See Badunenko and Kumbhakar (2020) for a discussion of different generations of SF models.

[6] For the EKC to be an inverted U-shape, β_2 needs to be negative.

[7] Several papers have posited and estimated cubic relationships, giving rise to N-shaped EKCs (Galeotti et al., 2006; Shahbaz and Sinha, 2019) or even inverted-M (or W) shaped EKCs (Yang et al., 2015; Hasanov et al., 2021). We did not consider such possibilities as the focus here is on developing a new approach for the conventional inverted-U shaped EKC.

(5) $$h_{it}(lny_{it};\beta_1,\beta_2) = \begin{cases} \frac{1}{\sqrt{1+(\beta_1+2\beta_2 lny_{it})^2}} & \text{for } lny_{it} < -\frac{\beta_1}{2\beta_2} \\ 1 & \text{otherwise} \end{cases}$$

Note that even if the vertical distance u_i is time-invariant, the emissions inefficiency u_{it} is time- and country-specific and will depend on the gap factor h_{it}. We choose this scaling formulation since it adds some useful dimensions to the framework. More specifically, it allows country heterogeneity to show up by shrinking or keeping the same inefficiency distribution without changing its basic shape. We also note that u_{it} will be time-invariant past the turning point. Following the bulk of the SFA literature, the inefficiency term is assumed to be half-normally distributed, $u_i \sim N^+\left(0, \sigma_{u_i}^2\right)$, and the idiosyncratic term is assumed to be normally distributed, $v_{it} \sim N(0, \sigma_v^2)$.[8]

Since in this paper we deal with panel data, it is important to account for heterogeneity among countries. This aspect is not considered in the second-generation models. One way to do this is to include country dummy variables, which can result in an incidental parameter problem described by Greene (2005). Another way is to include many time-constant variables that define differences in countries. However, it will be difficult in any given sample to identify which variables are required to fully account for unobserved heterogeneity. Besides, panel data often contain unobserved heterogeneity which may not be possible to model. In such cases, country-specific effects are added to the basic model in (4), so that:

(6) $$lne_{it} = f(\cdot) + \omega_i + v_{it} + u_{it}$$

where ω_i is a country-specific effect. Specification (6) is known as the third-generation stochastic frontier model. The term ω_i has been interpreted differently in the literature. For example, Kumbhakar and Hjalmarsson (1993, 1995) and Kumbhakar and Heshmati (1995) have estimated the model in (6) assuming that ω_i is the persistent or time-constant inefficiency. In this case, equation (6) becomes:

[8] We will maintain the assumption that u_i is heteroskedastic . Further details of estimation of the model in (3) are provided in Appendix.

(7) $$lne_{it} = f(\cdot) + u_{0i} + v_{it} + u_{it}$$

where observations are assumed to have two types of inefficiencies, namely the transient or short-term inefficiency $u_{0i} > 0$ and the persistent or long-term inefficiency $u_{it} > 0$. The interpretation of persistent inefficiency is that it is structural and cannot be changed over time. This fits poorly within our framework, where we wish to show that emissions inefficiency is based on the country's economic development or the gap to the turning point measured by the h_{it}, which can change over time.[9]

Greene (2005), on the other hand, has assumed that ω_i is an individual effect as we know it from standard panel data approaches. Hence the model (6) becomes:

(8) $$lne_{it} = f(\cdot) + v_{0i} + v_{it} + u_{it}$$

where $v_{0i} \sim N\left(0, \sigma_{v_0}^2\right)$ is a symmetric country-specific effect that can be both positive and negative. Model (8) is chosen over model (7) for two reasons. First, it is close in spirit to models currently employed to estimate a turning point for an EKC (Shuai et al., 2017). Second, as previously argued, we are attempting to measure the environmental inefficiency that depends on the time-varying economic development. It is tempting to make use of the fourth-generation stochastic frontier model which combines both the unobserved heterogeneity as in (8) and time-constant inefficiency as in (7) (Filippini and Hunt, 2016). However, as we argued before, this would not be consistent with our framework.

Therefore, we estimate (8) using the maximum simulated likelihood method (the details are given in the Appendix). The panel-level simulated log-likelihood contribution for ith observation is given as:

(9) $$\ln L_i^S(\boldsymbol{\theta}) = \ln\left\{\frac{1}{R}\sum_{r=1}^{R}\left[\prod_{t=1}^{T_i}\left(\frac{2\sigma_{*i}}{(2\pi)^{T_i/2}\sigma_v^{T_i}\sigma_{u_i}}\exp\left(-\frac{1}{2}a_{*ir}\right)\Phi\left(\frac{\mu_{*ir}}{\sigma_{*i}}\right)\right)\right]\right\}$$

[9] The derivative of (6) with respect to economic development is negative meaning that h_{it} is decreasing with economic development.

where $\sigma_{*i} = \sqrt{\frac{\sigma_v^2 \sigma_{u_i}^2}{\sigma_v^2 + \sigma_{u_i}^2 \sum \mathbf{h}_i^2}}$, $a_{*ir} = \frac{\sum \boldsymbol{\varepsilon}_{ir}^2}{\sigma_v^2} - \frac{\mu_{*ir}^2}{\sigma_{*i}^2}$, $\mu_{*ir} = \frac{\sigma_{*i}^2}{\sigma_v^2} \sum \mathbf{h}_i \boldsymbol{\varepsilon}_{ir}$, $\boldsymbol{\varepsilon}_{ir} = (\varepsilon_{i1r}, \dots, \varepsilon_{iT_ir})$, $\boldsymbol{h}_i = (h_{i1}, \dots, h_{iT_i})$, $\varepsilon_{itr} = lne_{it} - f(\cdot) - V_{0ir}\sigma_{v_0}$, and V_{0ir} is the random deviate from a standard normal distribution and R is the number of Monte-Carlo replications to approximate the simulated log-likelihood function in (9).[10] The log-likelihood for the whole sample is the sum of the logs of the panel level likelihoods $\ln L_i^S(\boldsymbol{\theta})$ defined in (9):

$$\ln^S L(\boldsymbol{\theta}) = \sum_{i=1}^N \ln L_i^S(\boldsymbol{\theta}) \tag{10}$$

After obtaining the estimates of the frontier and variance components, the estimator of the inefficiency can be approximated using Monte-Carlo integration:

$$\hat{E}^S[u_i|\text{data}] = \frac{1}{R}\sum_{r=1}^R w_{ir}\left\{\mu_{*ir} + \sigma_{*i}\frac{\phi\left(\frac{\mu_{*ir}}{\sigma_{*i}}\right)}{\Phi\left(\frac{\mu_{*ir}}{\sigma_{*i}}\right)}\right\} \tag{11}$$

where $w_{ir} = \frac{L_{ir}^S(\boldsymbol{\theta}^*)}{\frac{1}{R}\sum_{r=1}^R L_{ir}^S(\boldsymbol{\theta}^*)}$ and $\ln_{ir}^S(\boldsymbol{\theta}^*)$ is the likelihood for i and r evaluated at the optimal vector of parameters $\boldsymbol{\theta}^*$, which can be technically obtained in the last iteration of the maximum simulated likelihood optimization (we provide more details in the Appendix). Since the quantity in (11) provides an estimate of the vertical distance u_i, the emissions efficiency estimator is the exponent of the negative quantity in (11) multiplied by $\mathbf{h}_i$.

3.6 The role of environmental policy

In our flexible framework, we allow the vertical distance and hence emissions inefficiency (which in our empirical application below is carbon inefficiency) to be explained by an additional variable that does not affect the frontier shown in (4). As mentioned in the introduction, we assume that environmental policy fosters efficiency improvements in the emission intensity for given levels of GDP per capita. We assume that the u_i term is heteroskedastic with a variance $\sigma_{u_i}^2 = \exp\left[\frac{1}{2}(\delta_0 + \delta_1 EPS_i)\right]$,[11] where EPS_i is a country specific

[10] Full details of derivation are provided in the appendix.
[11] Exponentiation is applied to ensure positive variance.

environmental policy stringency and where we expect $\delta_1 < 0$.[12] The change in inefficiency prompted by a change in the environmental policy variable while holding everything else fixed is given by:

$$(12) \qquad \frac{\partial u_i}{\partial EPS_i} \approx \frac{\partial E[u_i]}{\partial EPS_i} = \sqrt{\frac{2}{\pi}} \frac{\partial \sigma_{u_i}}{\partial EPS_i}.$$

The latter equality follows from the assumption that u_i is half-normally distributed, whereby the expected value of u_i is equal to $\sqrt{2/\pi}\,\sigma_{u_i}$. Then (12) becomes: [13]

$$(13) \qquad \frac{\partial u_i}{\partial EPS_i} \approx \frac{1}{\sqrt{2\pi}} \delta_1 \exp\left[\frac{1}{2}(\delta_0 + \delta_1 EPS_i)\right]$$

Using our main specification (4) where the frontier does not depend on EPS, the marginal effect of environmental policy stringency on per capita (log) emissions can be computed as follows:

$$(14) \qquad \frac{\partial lne_{it}}{\partial EPS_i} = h_{it} \frac{\partial u_i}{\partial EPS_i} \approx [h_{it}] \times \left\{\frac{1}{\sqrt{2\pi}} \delta_1 \exp\left[\frac{1}{2}(\delta_0 + \delta_1 EPS_{it})\right]\right\}$$

Emissions in log per capita terms (which our empirical application below is CO_2 in log per capita terms) are reduced by an increase in environmental policy stringency. Note that, however, the effect is reduced by being to the left of the turning point where $h_{it} < 1$. Finally, the reduction in lne can be thought of as a rate of change, since Δlne is approximately equal to $(lne_1/lne_0) - 1$. Hence, expression (14) multiplied by 100 gives the percentage reduction in e due to a change in the policy index by one.

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[12] As shown below, the log-level specification provides an interesting interpretation of the outcome. An increase in *EPS* by 1 leads to a percentage change in the left -hand side outcome variable. Since the whole effect depends not only on δ_1, but also on the level of *EPS*, this specification enables us to obtain quite a flexible country-specific interpretation.

[13] To compute (13) note that $\sigma_{u_i}^2 = \exp\left[\frac{1}{2}(\delta_0 + \delta_1 EPS_i)\right]$. Thus, taking the derivative with respect to *EPS* - see (12) - we have: $\sqrt{\frac{2}{\pi}} \frac{\partial \sigma_{u_i}}{\partial EPS_i} \approx \sqrt{\frac{2}{\pi}} \frac{1}{2} \delta_1 \exp\left[\frac{1}{2}(\delta_0 + \delta_1 EPS_i)\right]$.

4. Data

We use annual data to implement our SEKF econometric model. Data on CO_2 emissions from fuel combustion (in Metric tons) are taken from the Global Energy & CO2 Database of Enerdata and a panel is compiled for the OECD countries spanning the period 1970–2018. In practice, for the variables of interest the data on the initial year of the sample vary across countries, so that the panel is unbalanced. We report the actual sample size and initial and final years of data in Table 1. Column T_i shows the number of observations for a country. In addition, due to the lack of sufficient data for the present investigation, we omitted Chile, Estonia, Iceland, Israel, Latvia, Luxembourg, Mexico, New Zealand, and Slovenia. The final sample consisted of 26 OECD countries.

Table 1: Sample used

		Year	
Country	T_i	Min	Max
Australia	40	1978	2017
Austria	40	1978	2017
Belgium	38	1980	2017
Canada	38	1980	2017
Czech Republic	25	1993	2017
Denmark	40	1978	2017
Finland	40	1978	2017
France	40	1978	2017
Germany	38	1980	2017
Greece	28	1990	2017
Hungary	26	1992	2017
Ireland	38	1980	2017
Italy	38	1980	2017
Japan	38	1980	2017
Netherlands	40	1978	2017
Norway	40	1978	2017
Poland	28	1990	2017
Portugal	38	1980	2017
Slovakia	25	1993	2017
South Korea	38	1980	2017
Spain	38	1980	2017
Sweden	38	1980	2017
Switzerland	38	1980	2017
Turkey	40	1978	2017
United Kingdom	38	1980	2017
United States	38	1980	2017

Besides CO_2 emissions, the other key variable is real GDP at constant purchasing power parity (PPP), expressed in millions 2015 U.S. dollars. To avoid scale effects, both emissions and GDP are converted to per capita terms (by dividing them by population, expressed in thousand individuals) and their descriptive statistics are presented in Table 2. Additionally, Table 2 shows the years in which the minimum and maximum values are observed for each country.

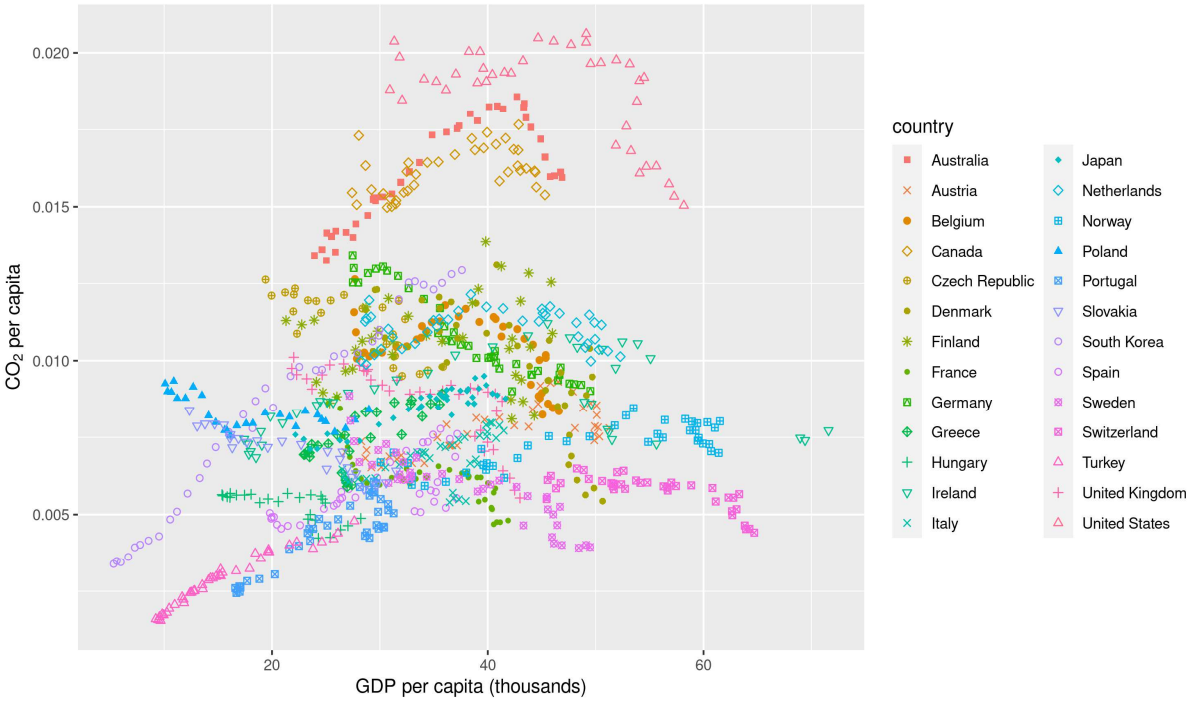
Table 2: Descriptive statistics for per capita CO_2 emissions and GDP

	CO_2 per capita					GDP per capita				
Country	Min	Year	Max	Year	Mean	Min	Year	Max	Year	Mean
Australia	0.013	1983	0.019	2007	0.016	23.941	1978	46.874	2017	35.085
Austria	0.007	1982	0.009	2005	0.008	26.792	1978	51.274	2017	40.281
Belgium	0.008	2014	0.013	1980	0.010	27.606	1981	46.597	2017	37.899
Canada	0.015	1986	0.018	2007	0.016	27.419	1982	45.300	2017	36.545
Czech Republic	0.010	2014	0.013	1993	0.011	19.402	1993	35.855	2017	27.522
Denmark	0.005	2017	0.013	1996	0.010	28.689	1978	50.646	2017	41.063
Finland	0.008	2015	0.014	2003	0.011	21.297	1978	45.992	2008	34.525
France	0.005	2014	0.009	1979	0.006	25.195	1978	41.882	2017	34.557
Germany	0.009	2009	0.013	1980	0.011	27.472	1980	49.508	2017	38.274
Greece	0.006	2016	0.009	2007	0.007	22.963	1993	35.752	2007	28.154
Hungary	0.004	2013	0.006	1996	0.005	15.368	1993	28.231	2017	21.458
Ireland	0.007	1984	0.011	2001	0.009	17.421	1980	71.586	2016	38.346
Italy	0.005	2014	0.008	2004	0.007	26.522	1980	41.476	2007	35.383
Japan	0.007	1982	0.009	2013	0.008	22.173	1980	41.651	2017	34.118
Netherlands	0.010	1983	0.012	1996	0.011	28.253	1982	52.289	2017	40.167
Norway	0.006	1983	0.008	1999	0.007	30.061	1978	61.517	2007	48.955
Poland	0.008	2002	0.009	1990	0.008	10.093	1991	28.985	2017	18.233
Portugal	0.002	1980	0.006	2002	0.004	16.588	1984	31.276	2017	25.333
Slovakia	0.006	2014	0.008	1993	0.007	12.347	1993	31.506	2017	21.600
South Korea	0.003	1980	0.013	2017	0.008	5.320	1980	37.603	2017	20.564
Spain	0.005	1985	0.008	2005	0.006	19.800	1981	37.163	2007	29.437
Sweden	0.004	2015	0.009	1980	0.006	27.177	1980	49.479	2017	37.768
Switzerland	0.004	2017	0.006	1985	0.006	45.454	1982	64.697	2017	55.371
Turkey	0.002	1979	0.005	2017	0.003	9.224	1980	27.629	2017	15.598
United Kingdom	0.006	2017	0.010	1980	0.009	21.838	1981	42.985	2017	33.496
United States	0.015	2017	0.021	2000	0.019	30.923	1982	58.174	2017	45.550

Focussing on the mean values, it can be seen that the countries with the highest per capita emissions are U.S., Australia, and Canada, respectively. At the opposite end is Turkey, Portugal, and Hungary. When it comes to per capita GDP, Switzerland, Norway, and the U.S. are the richest countries; Turkey, Poland, and Hungary are the less rich ones. One interesting indication that can be drawn from the table is how early the maximum per capita emissions level occurred: 1979 for France and 1980 for Belgium, Germany, Sweden, and the U.K. Year 2017 was the year when the lowest level of per capita emissions was reached in Denmark, Switzerland, U.K., and

the U.S. Figure 7 shows the scatter plot of per capita emissions vs per capita GDP and the pattern appears to be compatible with an inverted U-shape relationship.

Figure 7: Scatter plot of per capita CO_2 emissions vs per capita GDP

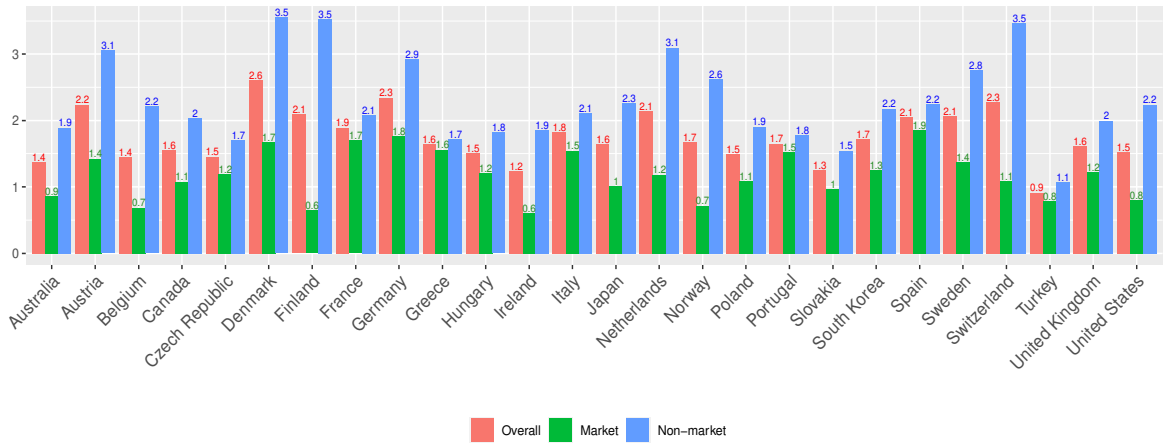


Additional control variables that proved to be significant in estimation are the share of industry value added in total GDP, the price of gasoline (premium gasoline in 2015 PPP U.S. dollars), and population density (people per squared kilometer). Data for all these variables are taken from Enerdata.

The indicator of environmental policy stringency used is the OECD Environmental Policy Stringency (*EPS*) (Botta and Koźluk, 2014). The EPS database contains information on 15 different Non-Market-Based (*EPS-NMKT*) and Market-Based (*EPS-MKT*) environmental policy instruments implemented in OECD countries. *NMKT* policies include limits to pollutants (SO_x, NO_x, Particulate Matters and Sulphur Content of Diesel) and government energy-related R&D expenditures as a percentage of GDP. *MKT* policies include feed in tariffs (FIT) for solar and wind power, taxes (on CO_2, SO_x, NO_x and Diesel), certificates (White, Green and CO_2) and the presence of deposit and refund schemes (DRS). All variables in the database are continuous, except DRS which is a 0/1 indicator for the presence of such schemes. The main steps of the methodology used to compute the EPS indicator are the following (see, for details, Botta and

Koźluk, 2014). First, each of the continuous policy instruments of the database is categorized on a Likert scale from 0 to 6 using statistical procedures to identify specific bins. These 15 Likert-scale scores are then aggregated into 6 large macro-instruments: Taxes, Certificates, Limits, FIT, DRS and R&D by using weights. Subsequently, these 6 indicators are aggregated into an *MKT* score (Taxes, Certificates, FIT, DRS) and an *NMKT* score (R&D and Limits). The EPS composite score is then obtained as the average between the *MKT* and *NMKT* scores. Data for EPS are available for OECD countries annually from 1990 to 2012 or 2015 for selected countries. Table 3 presents the descriptive statistics of the policy indicator with each country ordered alphabetically.

Figure 8: Descriptive statistics of EPS environmental policy indicator



Note: Average values over the period 1990-2012. Red, green, and blue bars refer to overall *EPS*, *EPS-MKT*, and *EPS-NMKT* policy indicators.

Figure 8 shows that the value of the indicator for non-market policies is systematically higher than that referred to market policies. This evidence appears in line with the fact that incentive-based instruments have been adopted later in time than non-market-based instruments, these ones being traditionally been more familiar to bureaucratic apparatuses. As shown in Figure 11 below, this situation has changed in more recent years.

5. Empirical results

We estimate model (8) using (4), which we report here:

(15) $$lne_{it} = \beta_0 + \beta_1 lny_{it} + \beta_2 (lny)_{it}^2 + \mathbf{x}_{it}\boldsymbol{\gamma} + v_{0i} + v_{it} + u_{it}$$

where $v_{0i} \sim N(0, \sigma_{v_0}^2)$ is a symmetric country-specific effect capturing unobserved heterogeneity and, $v_{it} \sim N(0, \sigma_v^2)$ is the idiosyncratic term. Finally, $u_{it} = h_{it} u_i > 0$ is the inefficiency term with $u_i \sim N^+(0, \sigma_{u_i}^2)$ assumed to be half-normally distributed. Note that the income turning point is given by $e^{-\beta_1/(2\beta_2)}$.

In Table 3 we contrast the standard EKC model based on a random effects specification (RE), where there is no u_{it} error component, with a stochastic frontier approach to the Kuznets relationship (SEKF).

Table 3: Random effects vs Stochastic environmental Kuznets frontier (SEKF) estimation results

Variable	RE	SEKF
Estimated coefficients		
Constant	−8.584 (−33.88)	−8.459 (−43.24)
ln(GDPpc)	2.079 (13.71)	1.675 (14.52)
ln(GDPpc) ^{2}	−0.269 (−10.87)	−0.220 (−12.03)
ln(trend)	−0.093 (−3.14)	−0.067 (−2.86)
Industry in GDP	0.568 (2.40)	0.793 (3.41)
ln(Gasoline price)	−0.238 (−8.82)	−0.257 (−10.49)
Population density	0.001 (2.15)	0.001 (2.30)
Variance of random components		
$\ln\sigma_v^2$	−2.222 (−7.75)	−3.954 (−84.69)
$\ln\sigma_{v_0}^2$	−4.026 (-87.51)	−12.294 (−0.02)
Variance of u_i*:* $ln\sigma_{u_i}^2$		
Constant		−1.612 (−7.77)
$\bar{\theta}$	0.934	
N	26	26

$\sum_{i=1}^{N} T_i$	972	972
lnL	507.2	475.8
Turning point	47.62	45.07
Lower Bound	36.49	36.59
Upper Bound	58.76	53.55

Notes: (i) z-statistics in round brackets; (ii) the EKC model is estimated using a random effects specification; the SEKF is based on a Stochastic frontier approach, where the vertical distance term (u_i) is homoscedastic; (iii) $\bar{\theta}$ is the average over N of θ_i, which is the familiar RE term $\theta_i = 1 - \sigma_v / \sqrt{\left(T_i \sigma_{v_0}^2 + \sigma_v^2\right)}$ (note that since $\bar{\theta}$ is very close to unity, the RE estimates are close to the FE estimates); (iv) the upper and lower bounds of the 95% confidence interval are calculated as the estimate of the turning point plus and minus 2 standard deviations of $e^{-\widehat{\beta}_1 / (2\widehat{\beta}_2)}$, which are estimated using the Delta method.

Both specifications are consistent with an inverted-U shaped relationship between per capita emissions and GDP. The estimated coefficients for squared GDP are statistically significant and negative, as expected. On the basis of these estimated parameters, it is possible to compute the implied per capita income turning point, as highlighted in Section 3.5 above. The value is between 47 and 45 thousand dollars for the two models: the lower value associated with the SEKF model appears to be more consistent with the values shown in Table 2. As we have argued earlier, the turning point is a concept rather than a precise estimate of income at which economy becomes industrialised. Hence the 95% confidence interval is calculated to account for sampling variation. The estimates suggest that the economy becomes industrialised somewhere between 36 and 53 thousands of US dollars in the 2015 prices.

Next, the basic EKC specification is augmented. Industry value added controls for the composition of GDP, as changes in the structure of GDP may account for the behaviour of emissions, besides the absolute level of GDP itself. Similar considerations apply for population density, which control for the spatial distribution of people, in addition to their sheer number. A time trend is added to capture the impact of country-invariant time-specific factors and the price of gasoline is a proxy for energy prices which may affect the composition of the energy mix, and in turn of carbon dioxide emissions. All these variables are statistically significant with the expected signs in both model specifications.[14]

Finally, all variances are statistically significant, especially the variance of the inefficiency term. We can now assess whether the distance of a country at a point in time from the efficiency frontier is or can be affected by environmental policy. To that end, it is assumed that the u_i term

[14] The empirical results from estimation of the standard EKC model with no additional controls, both in its RE and its SEKF versions are not shown here for space reasons. They are available from the authors upon request.

in (15) is heteroskedastic with a variance that depends on the environmental policy stringency indicator. Specifically, it is assumed that $\sigma_{u_i}^2 = \exp(\delta_0 + \delta_1 EPS_i + \delta_2 EPS\text{-}NMKT_i + \delta_3 EPS\text{-}MKT_i)$, where EPS_i is the country-specific OECD environmental policy stringency indicator and where we expect both $\delta_1 < 0$, $\delta_2 < 0$, and $\delta_3 < 0$. Table 4 presents the role of environmental policy stringency and its impact on inefficiency, where Model (a) corresponds to the case where $\delta_2 = \delta_3 = 0$, Model (b) where $\delta_1 = \delta_3 = 0$, Model (c) where $\delta_1 = \delta_2 = 0$, and Model (d) where $\delta_1 = \delta_1 = 0$ and both $\delta_2 \neq 0$ and $\delta_3 \neq 0$.

Table 4: Estimation results of the Stochastic environmental Kuznets frontier (SEKF) model with environmental policy stringency indicators

Variable	Model (a)	Model (b)	Model (c)	Model (d)
Estimated coefficients				
Constant	−8.466 (-43.02)	−8.492 (-42.68)	−8.458 (-43.06)	−8.492 (-42.61)
ln(GDPpc)	1.677 (14.39)	1.683 (14.32)	1.675 (14.44)	1.683 (14.29)
ln(GDPpc) ^{2}	−0.220 (-11.95)	−0.222 (-11.92)	−0.220 (-11.96)	−0.222 (-11.90)
ln(trend)	−0.068 (−2.90)	−0.065 (−2.77)	−0.068 (−2.89)	−0.065 (−2.77)
Industry in GDP	0.789 (3.38)	0.809 (3.47)	0.788 (3.38)	0.808 (3.47)
ln(Gasoline price)	−0.256 (-10.45)	−0.254 (-10.35)	−0.257 (-10.49)	−0.254 (-10.36)
Population density	0.001 (2.47)	0.001 (2.69)	0.001 (2.32)	0.001 (2.69)
Variance of random components				
$\ln\sigma_v^2$	−3.954 (-84.68)	−3.954 (-84.67)	−3.954 (-84.69)	−3.954 (-84.67)
$\ln\sigma_{v_0}^2$	-15.094 (-5.5e-3)	-17.936 (-2.8e-3)	-17.69 (-2.8e-3)	-14.503 (-7.3e-3)
Variance of u_i*:* $ln\sigma_{u_i}^2$				
Constant	−0.391 (−0.44)	−0.235 (−0.34)	−1.198 (−1.57)	−0.189 (−0.21)
EPS	−0.724 (−1.46)			
EPS-MKT		−1.239 (−2.18)		−1.223 (−2.05)
EPS-NMKT			−0.183 (−0.57)	−0.028 (−0.08)
N	26	26	26	26
$\sum_{i=1}^{N} T_i$	972	972	972	972
lnL	476.77	478.04	475.95	478.04

Turning point	45.01	44.19	45.18	44.21
Lower Bound	36.58	35.99	36.64	35.99
Upper Bound	53.42	52.39	53.72	52.44

Notes: (i) z-statistics in round brackets; (ii) The EPS indicators used here are the average value per country over the period 1990-2012.

The table shows that all explanatory variables included in all of the SEKF models are statistically significant and with the expected sign. Thus, the EKF is confirmed and the income turning point has values in line with the previous estimates in Table 3, with slightly lower levels for Models (b) and (d). The 95% confidence intervals for the turning point are quite similar across all estimated models suggesting the beginning of the major transformation of an economy at around 36 thousands US dollar of GDP per capita.

Focussing on the effect of environmental policy stringency on the degree of inefficiency, Table 4 shows that a significant negative impact is only found for market-based policy instruments. The coefficient of the overall *EPS* indicator in Model (a) is hardly significant, whereas that of the Non-Market-Based indicator (*EPS-NMKT*) is insignificant in Model (c) and Model (d). Our preferred model is therefore Model (b) which shows how climate policies such as carbon pricing measures, subsidies to clean energy sources and the like are potentially capable to reduce the distance of a country-time from the EKF. Hence, the remainder of our inference is based on Model (b).

Focussing now on the role of environmental policy stringency on inefficiency, Table 5 presents the estimated effect on CO_2 emissions. Recall from (14) that the effect of environmental policy stringency on (the log of) carbon dioxide emissions per capita is given by the product of the effect of the policy indicator on carbon inefficiency times the gap factor h, that is: $\frac{\partial lne_{it}}{\partial EPS_i} = h_i \frac{\partial u_{it}}{\partial EPS_i}$. The table therefore shows the overall marginal impact of the market-based environmental policy stringency indicator on emissions (columns 1-3), which is decomposed into policy enhancing effect through improvements in carbon inefficiency (column 4) and the restricting effect of being below the turning point which is represented by the estimated values of $\hat{h}$ which, when is equal to one (after the turning point), indicate full effect of policy on emissions (columns 5-7).

According to the gap factor in column 2, Table 5 shows that on average all countries were before the income turning point although generally very close to it. Only Switzerland has a h value of 1 both on average, but also as minimum and maximum values. The countries with a larger gap

between per capita GDP and the turning point were (on average) Turkey, South Korea, and Poland. These three countries, together with Slovakia, also show the minimum value of the gap factor, hence the biggest distance from the turning point recorded during the 1990-2012 period. The impact of policy stringency on carbon inefficiency, shown in column 4, is quantitatively very similar to the marginal effect on emissions, shown in columns 5-7. The estimated impacts show roughly the growth of emissions when *EPS-MKT* increases by one unit and the table shows that a reduction in emissions growth is generally equal to 0.2, with strongest average impacts for Ireland, Finland, Belgium, and Norway and weakest impacts for Spain, Germany, France, Denmark, and Portugal. These findings are also visualised in Figure 9 where the vertical axis represents average carbon inefficiency, the horizontal axis represents the average *EPS-MKT* index, and the scatter point size indicates the marginal effect of *EPS-MKT* on carbon inefficiency and emissions.

Table 5: Estimated effect of environmental policy stringency on CO_2 emissions

	Gap factor $\hat{h}$			Marginal effect on carbon inefficiency	Marginal effect on emissions		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Country	Min	Mean	Max		Min	Mean	Max
Australia	0.931	0.979	1.000	−0.257	−0.257	−0.252	−0.240
Austria	0.953	0.990	1.000	−0.182	−0.182	−0.180	−0.173
Belgium	0.958	0.989	1.000	−0.287	−0.287	−0.284	−0.275
Canada	0.957	0.988	1.000	−0.225	−0.225	−0.222	−0.215
Czech Republic	0.882	0.951	0.993	−0.210	−0.208	−0.200	−0.185
Denmark	0.964	0.993	1.000	−0.155	−0.155	−0.154	−0.150
Finland	0.905	0.977	1.000	−0.295	−0.295	−0.288	−0.267
France	0.941	0.983	1.000	−0.153	−0.153	−0.151	−0.144
Germany	0.957	0.990	1.000	−0.147	−0.147	−0.146	−0.141
Greece	0.922	0.957	0.991	−0.167	−0.166	−0.160	−0.154
Hungary	0.819	0.902	0.970	−0.208	−0.202	−0.188	−0.170
Ireland	0.854	0.959	1.000	−0.301	−0.301	−0.289	−0.257
Italy	0.951	0.987	0.999	−0.169	−0.169	−0.166	−0.160
Japan	0.914	0.981	1.000	−0.234	−0.234	−0.230	−0.214
Netherlands	0.962	0.990	1.000	−0.211	−0.211	−0.209	−0.203
Norway	0.972	0.997	1.000	−0.282	−0.282	−0.281	−0.274
Poland	0.699	0.850	0.974	−0.224	−0.219	−0.191	−0.157
Portugal	0.841	0.934	0.980	−0.171	−0.167	−0.159	−0.143
Slovakia	0.757	0.894	0.983	−0.242	−0.238	−0.216	−0.183
South Korea	0.530	0.844	0.996	−0.201	−0.201	−0.170	−0.107
Spain	0.887	0.959	0.996	−0.139	−0.139	−0.134	−0.123

Sweden	0.955	0.987	1.000	−0.188	−0.188	−0.185	−0.179
Switzerland	1.000	1.000	1.000	−0.225	−0.225	−0.225	−0.225
Turkey	0.674	0.809	0.960	−0.271	−0.260	−0.219	−0.182
United Kingdom	0.911	0.975	1.000	−0.206	−0.206	−0.201	−0.188
United States	0.975	0.996	1.000	−0.267	−0.267	−0.266	−0.261

Notes: (i) Calculations based on Model (b) in Table 4 for the Market-Based EPS (*EPS-MKT*) indicator; (ii) Columns 1 to 3 present the min, mean, and max values of the estimated gap factor h; (iii) Column 4 reports the marginal effect of *EPS-MKT* on carbon inefficiency as given by $\partial u_{it} / \partial EPS_i$; (iv) Columns 5 to 7 reports the marginal effect of *EPS-MKT* on (log) per capita emissions, as given by $\partial lne_{it} / \partial EPS_i$ in equation (14) in the main text.

Figure 9: Marginal effect of the environmental policy index EPS-MKT plotted against the average policy index

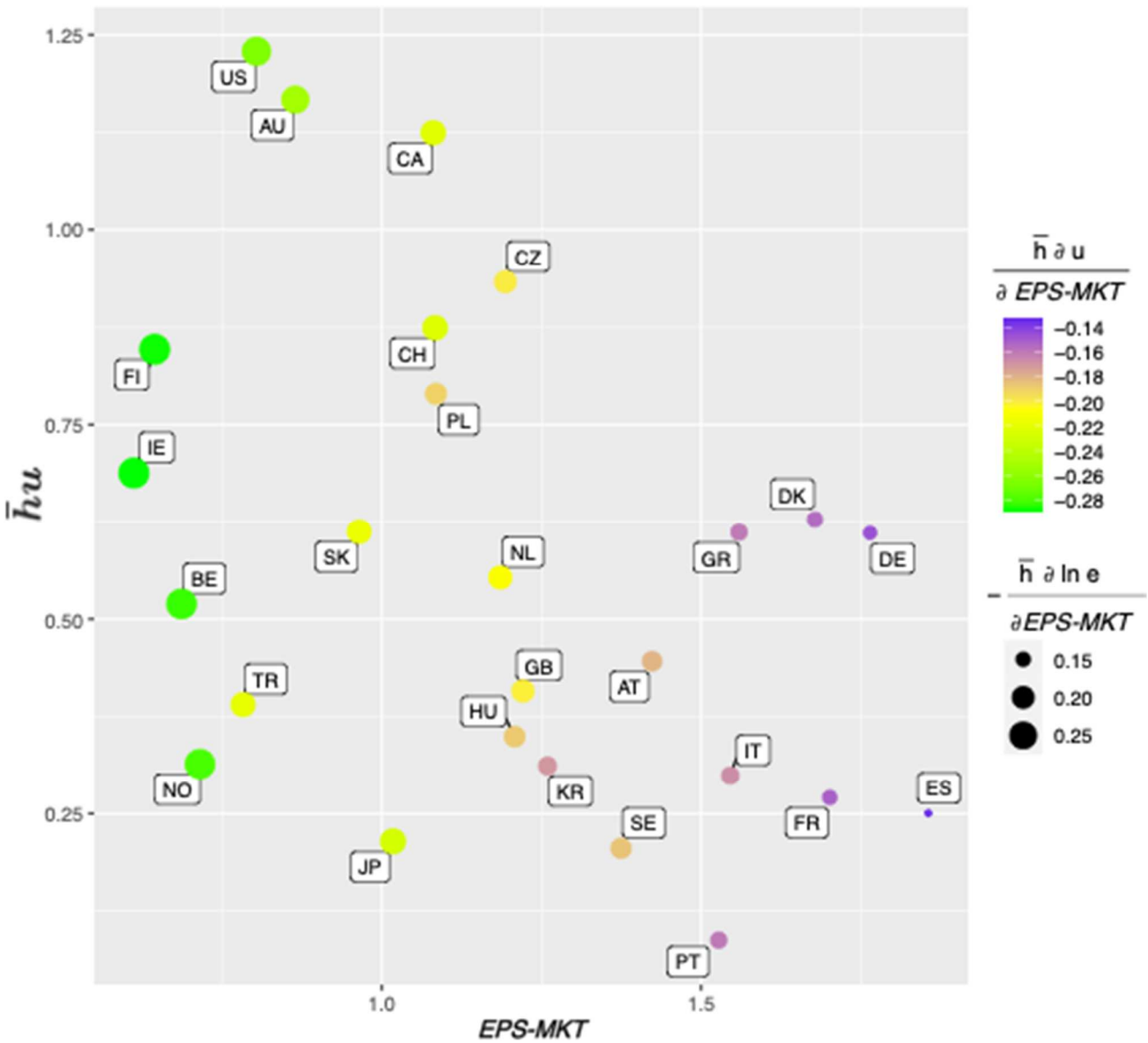


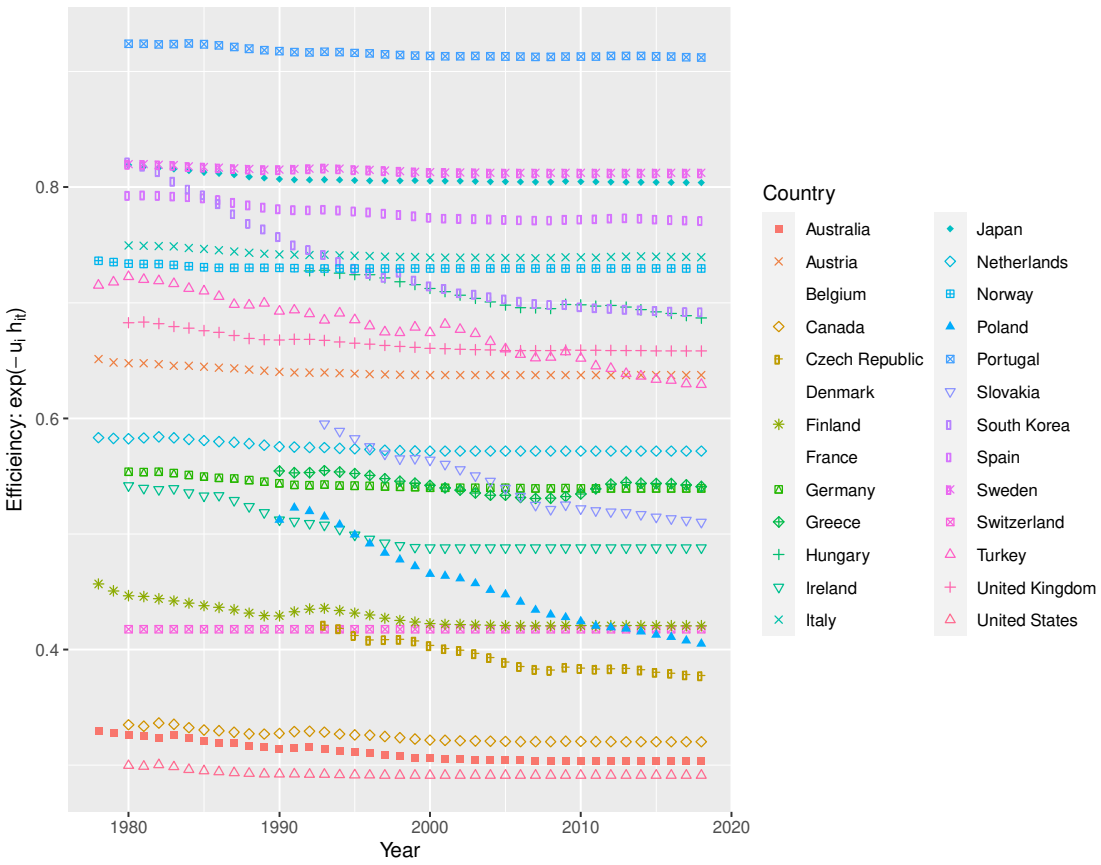
Table 6 presents the mean and the extreme values over the sample period of the estimated value of efficiency, as given by $e^{-\hat{h}_{it}\hat{E}^{S}[u_i|\text{data}]}$. South Korea, Sweden, and Japan are shown to be the most efficient during the period. At the opposite extreme, U.S., Australia, and Canada are shown to be the least efficient as well as showing very little improvement in their efficiency scores given the similarities in their minimum and maximum values.

Table 6: Estimated carbon efficiency

Country	Min	Mean	Max
Australia	0.304	0.311	0.330
Austria	0.637	0.640	0.651
Belgium	0.591	0.595	0.605
Canada	0.320	0.325	0.336
Czech Republic	0.378	0.394	0.421
Denmark	0.531	0.534	0.543
Finland	0.421	0.429	0.457
France	0.759	0.763	0.772
Germany	0.539	0.543	0.554
Greece	0.531	0.543	0.555
Hungary	0.687	0.705	0.728
Ireland	0.488	0.503	0.542
Italy	0.739	0.742	0.750
Japan	0.804	0.807	0.819
Netherlands	0.572	0.575	0.584
Norway	0.730	0.730	0.736
Poland	0.405	0.456	0.523
Portugal	0.912	0.916	0.924
Slovakia	0.510	0.543	0.595
South Korea	0.692	0.733	0.822
Spain	0.771	0.779	0.793
Sweden	0.812	0.815	0.820
Switzerland	0.417	0.417	0.417
Turkey	0.629	0.677	0.723
United Kingdom	0.658	0.665	0.683
United States	0.291	0.293	0.300

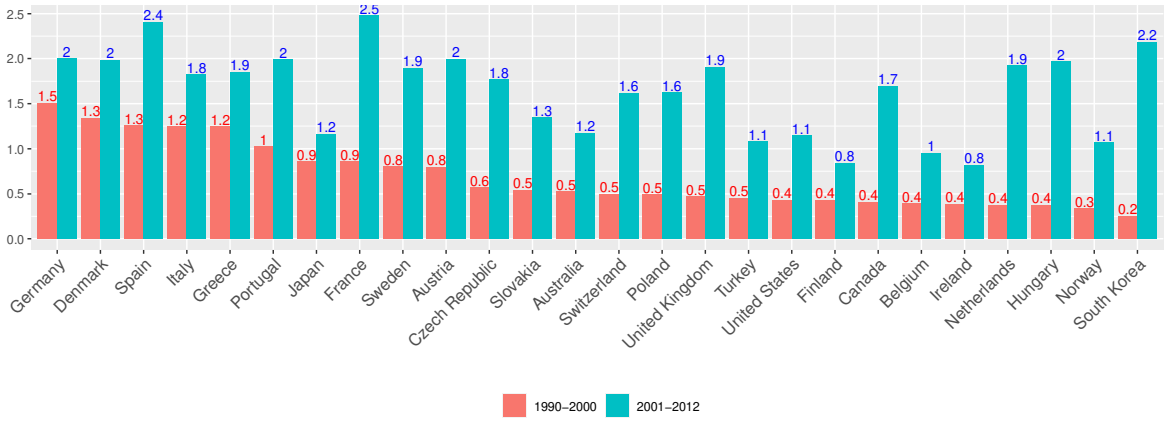
Figure 10 shows the estimated value of efficiency by country and its evolution over time. It shows that for several countries, efficiency was relatively stable over time, the exception being the marked reduction by South Korea, Turkey, Slovakia, and Poland.

Figure 10: Estimated carbon efficiency by country over time



So far, the analysis has considered the impact of environmental policy on efficiency and emissions by looking at the average over the whole sample period 1990-2012. However, environmental policy has generally become more stringent over time. To give a sense of this tendency the share of global greenhouse gas emissions covered by carbon taxes and emission trading systems, according to the World Bank (2021), was 2% in 1990 and 64% in 2021. This is confirmed by the *EPS-MKT* indicator: when the sample is split between the first decade 1990-2000 and the second one 2001-2012, Figure 11 shows that in every country policy action became stronger. Given this, Model (b) from Table 4 was re-estimated with the *EPS-MKT* policy indicator split into these two sub-periods with results for the two estimated SEKFs shown in Table 7.

Figure 11: Market-based environmental policy indicator EPS-MKT for sub-periods 1990-2000 and 2001-2012



Note: the *EPS-MKT* indicator is sorted by its 1990-2000 value (brown bars).

Table 7: Estimation results of the Stochastic environmental Kuznets frontier (SEKF) model with market-based environmental policy stringency indicator split by sub-samples

Variable	Model (e)	Model (f)
Estimated coefficients		
Constant	−8.467	−8.511
	(-43.03)	(-41.46)
ln(GDPpc)	1.681	1.682
	(14.49)	(13.89)
ln(GDPpc) ^{2}	−0.221	−0.223
	(-12.02)	(-11.52)
ln(trend)	−0.067	−0.062
	(−2.89)	(−2.59)
Industry in GDP	0.783	0.839
	(3.36)	(3.61)
ln(Gasoline price)	−0.257	−0.253
	(-10.44)	(-10.29)
Population density	0.001	0.001
	(2.31)	(3.03)
Variance of random components		
$\ln\sigma_v^2$	−3.954	−3.954
	(-84.68)	(-84.65)
$\ln\sigma_{v_0}^2$	-14.521	-11.470
	(-6.8e-3)	(−0.03)
Variance of u_i*:* $ln\sigma_{u_i}^2$		
Constant	−1.097	0.155

	(−2.70)	(0.18)
$EPS\text{-}MKT_{1990\text{-}2000}$	−0.818	
	(−1.60)	
$EPS\text{-}MKT_{2001\text{-}2012}$		−1.137
		(−2.15)
N	26	26
$\sum_{i=1}^{N} T_i$	972	972
lnL	476.93	478.2
Turning point	44.99	43.45
Lower Bound	36.53	35.2
Upper Bound	53.46	51.71

Notes: (i) z-statistics in round brackets; (ii) The EPS indicators used here refer to the average value over the sub-periods 1990-2000 and 2001-2012 respectively.

The results in Table 7 confirm the statistical significance and hence the relevance of the impact of market-based environmental policies on efficiency and emissions for the second decade of our sample, beginning in 2001. Indeed, during the last decade of the 1990s the role of environmental policy was weaker. Note that the income turning points are slightly lower than before.

We conclude the illustration of the empirical results by presenting the country-by-country marginal impact of environmental policy on carbon dioxide emissions distinguishing the policy action between the early and later periods reported in Table 8. This shows that for nearly all countries the impact becomes stronger in the second period relative to the first one. The exceptions are France, Hungary, and South Korea. For several countries the impact of environmental policy gets much stronger in the second period, as in the case of Australia, Belgium, Finland, Ireland, Japan, Norway, Slovakia, Turkey, and U.S.

Table 8: Estimated effect of market-based environmental policy stringency on emissions

Country	Marginal effect on emissions	
	1990-2000	2001-2012
Australia	−0.152	−0.251
Austria	−0.136	−0.157
Belgium	−0.161	−0.285
Canada	−0.160	−0.187
Czech Republic	−0.149	−0.180
Denmark	−0.109	−0.158
Finland	−0.158	−0.304

France	−0.133	−0.120
Germany	−0.102	−0.157
Greece	−0.113	−0.171
Hungary	−0.162	−0.160
Ireland	−0.161	−0.308
Italy	−0.113	−0.174
Japan	−0.133	−0.253
Netherlands	−0.162	−0.164
Norway	−0.165	−0.268
Poland	−0.154	−0.194
Portugal	−0.124	−0.158
Slovakia	−0.151	−0.228
South Korea	−0.170	−0.142
Spain	−0.113	−0.125
Sweden	−0.136	−0.167
Switzerland	−0.154	−0.195
Turkey	−0.157	−0.265
United Kingdom	−0.156	−0.166
United States	−0.158	−0.256

6. Summary and Conclusion

The standard approach to the Environmental Kuznets Curve (EKC) holds that as a country develops and GDP per capita grows there is an initial increase in emissions but eventually it will reach a point where economic and technological transformation will induce a decline in emissions. The EKC will exhibit an inverted U-shape suggesting that the main way for a country to reduce emissions is to continue to '*grow*'. However, this implicitly assumes that the country is on the EKC, whereas in reality a country, for various reasons, might be emissions inefficient and thus emitting above the best attainable level. In this case emissions could be reduced before and after the EKC by becoming more emissions efficient – i.e., to '*improve*'. In this paper we proposed and estimated an Environmental Kuznets Frontier (EKF) to represent the 'best' EKC across a number of OECD countries in order to benchmark each country against. Thus, giving an indication of how a country could '*grow*' and/or '*improve*' to reduce emissions.

To achieve this, we introduced the new concept of a Stochastic Environmental Kuznets Frontier (SEKF) and developed a framework that allows us to empirically analyse both solutions to reduce a country's emissions, that is via economic growth or through an improvement in emissions efficiency. In addition, we analysed the role and the stringency of environmental policies in reducing a country's emissions inefficiency measured by the distance from the benchmark EKF. Such emission reductions brought about by the re-organization of production and distribution within and outside the firm, changes in the energy mix, energy conservation

and behavioural changes toward energy savings are all cases where in principle it is possible to become more efficient at unchanged GDP. All these changes are likely to be policy-induced, which we explored via the introduction of an environmental policy stringency measure as a driver countries' emissions inefficiency.

Using this new approach, we estimated a SEKF using a cross-country analysis for the relatively homogenous group represented by OECD countries. The results support the idea of a benchmark inverted-U shaped EKF. The estimated turning point of per capita GDP is quite reasonable indicating that countries that 'grow' beyond the turning point would then reduce their carbon emissions. We also estimated carbon efficiency to be in the range from 30% (U.S.) to 82% (Sweden) and 92% (Portugal). This implies that much can be done to reduce emissions by 'improving' even at current economic development by reducing their carbon inefficiency. To see the determinants of carbon efficiencies, we then assessed whether the distance of a country at a point in time from the EKF as well as the emissions are or can be affected by environmental policy, which we measured using an indicator of environmental policy stringency (EPS). EPS is a well-known index provided by the OECD which comprises of both market-based and non-market-based policy instruments. However, we find support only for the impact of market-based environmental policy instruments given the coefficient on the market-based policy instrument was negative and statistically significant whereas the non-market-based policy instrument was always statistically insignificant. Our preferred model therefore indicates that climate policies such as carbon pricing measures, subsidies to clean energy sources and the like are potentially capable of helping to reduce the distance of a country-time from the efficiency frontier thereby reducing emissions.

We find that, when the environmental policy indicator goes up by 1 unit (the index ranges from 0 to 6), emissions growth falls, on average, by nearly 20%, with strongest average impacts for Ireland, Finland, Belgium, and Norway and weakest impacts for Spain, Germany, France, Denmark, and Portugal. Moreover, we find that environmental policy to curb carbon dioxide emissions becomes more stringent over time; when the sample is split between the first decade 1990-2000 and the second one 2001-2012, we find that in every country policy action becomes stronger. Indeed, for nearly all countries the impact becomes stronger in the second period relative to the first one.

We believe that the new approach introduced in this paper opens up an interesting line of research and we look forward to EKFs being estimated in future research studies. In particular, it would be good to see the approach applied to different data sets with different groups of

countries (such as a panel of developing countries), different environmental degradation indicators, and/or the use of alternative environmental policy indicators to explain emission inefficiency. Furthermore, the new approach introduced here applies to the conventional inverted-U (quadratic) shaped environmental Kuznets relationship; however, more recent papers have attempted to estimate N-shaped (cubic) and even inverted-M shaped (or W) shaped (quartic) environmental Kuznets relationships. Future research should therefore adapt and develop the technique introduced here to enable the estimation of N-shaped and inverted-M shaped EKFs.

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Appendix

1. The estimation of the model in (4)

To understand the derivation of the model in (8) the derivation of the model in (4) is shown first. The inefficiency term is assumed to be half-normally distributed, $u_i \sim N^+(0, \sigma_{u_i}^2)$. The idiosyncratic term is assumed to be normally distributed, $v_{it} \sim N(0, \sigma_v^2)$, and its density for each i is given by:

(A.1) $$f_v(v) = \frac{1}{\sqrt{2\pi}\sigma_v} \exp\left(-\frac{v^2}{2\sigma_v^2}\right)$$

Since $\mathbf{v} = (v_1, \dots, v_{T_i})$ is T_i–dimensional, the panel version of (A.1) is given by:

(A.2) $$f_v(\mathbf{v}) = \frac{1}{(2\pi)^{T_i/2}\sigma_v^{T_i}} \exp\left(-\frac{\mathbf{v}'\mathbf{v}}{2\sigma_v^2}\right)$$

Given the assumption of independence between u and $\mathbf{v}$, the joint density of u and $\mathbf{v}$ is simply the product of their density functions:

(A.3) $$f(u, \mathbf{v}) = \frac{1}{(2\pi)^{(T_i+1)/2}\sigma_v^{T_i}\sigma_{u_i}} \exp\left(-\frac{u^2}{2\sigma_{u_i}^2} - \frac{\mathbf{v}'\mathbf{v}}{2\sigma_v^2}\right)$$

Because $\varepsilon_{it} = v_{it} + u_{it} = v_{it} + h_{it}u_i$, we can write vector-wise for each panel $\boldsymbol{\varepsilon}_i = \mathbf{v}_i + \mathbf{h}_i u_i$, where $\mathbf{h}_i = (h_{i1}, \dots, h_{iT_i})$. Note that $\boldsymbol{\varepsilon}_i$, $\mathbf{v}_i$, and $\mathbf{h}_i$ are T_i-dimensional vectors, while u_i is a scalar. The joint density of u_i and $\boldsymbol{\varepsilon}_i$ for each panel is given by:

(A.4) $$f(u_i, \boldsymbol{\varepsilon}_i) = \frac{2}{(2\pi)^{(T_i+1)/2}\sigma_v^{T_i}\sigma_{u_i}} \exp\left(-\frac{u_i^2}{2\sigma_{u_i}^2} - \frac{(\boldsymbol{\varepsilon}_i - \mathbf{h}_i u_i)'(\boldsymbol{\varepsilon}_i - \mathbf{h}_i u_i)}{2\sigma_v^2}\right)$$

The marginal density of $\boldsymbol{\varepsilon}_i$ is obtained by integrating u_i out of the joint density in (A.4). To integrate u_i out, note that:

$$-\frac{u_i^2}{\sigma_{u_i}^2} - \frac{(\boldsymbol{\varepsilon}_i - \mathbf{h}_i u_i)'(\boldsymbol{\varepsilon}_i - \mathbf{h}_i u_i)}{\sigma_v^2} = -\frac{u_i^2}{\sigma_{u_i}^2} - \frac{\sum_{t=1}^{T_i}(\boldsymbol{\varepsilon}_i - \mathbf{h}_i u_i)^2}{\sigma_v^2}$$

$$= -\frac{u_i^2}{\sigma_{u_i}^2} - \frac{\sum \boldsymbol{\varepsilon}_i^2 - 2u_i \sum \mathbf{h}_i \boldsymbol{\varepsilon}_i + u_i^2 \sum \mathbf{h}_i^2}{\sigma_v^2}$$

$$= -\frac{\sigma_{u_i}^2\left(\sum \boldsymbol{\varepsilon}_i^2 - 2u_i \sum \mathbf{h}_i \boldsymbol{\varepsilon}_i + u_i^2 \sum \mathbf{h}_i^2\right) + \sigma_v^2 u_i^2}{\sigma_v^2 \sigma_{u_i}^2}$$

$$= -\frac{u_i^2\left(\sigma_v^2 + \sigma_{u_i}^2 \sum \mathbf{h}_i^2\right) - 2u_i \sigma_{u_i}^2 \sum \mathbf{h}_i \boldsymbol{\varepsilon}_i}{\sigma_v^2 \sigma_{u_i}^2} - \frac{\sum \boldsymbol{\varepsilon}_i^2}{\sigma_v^2}$$

Define $\sigma_{*i}^2 = \frac{\sigma_v^2 \sigma_{u_i}^2}{\sigma_v^2 + \sigma_{u_i}^2 \sum \mathbf{h}_i^2}$ and $\mu_{*i} = \frac{\sigma_{u_i}^2 \sum \mathbf{h}_i \boldsymbol{\varepsilon}_i}{\sigma_v^2 + \sigma_{u_i}^2 \sum \mathbf{h}_i^2} = \frac{\sigma_{*i}^2}{\sigma_v^2} \sum \mathbf{h}_i \boldsymbol{\varepsilon}_i$. Then the term in parentheses in (A.4) becomes:

$$-\frac{u_i^2}{\sigma_{u_i}^2} - \frac{(\boldsymbol{\varepsilon}_i - \mathbf{h}_i u_i)'(\boldsymbol{\varepsilon}_i - \mathbf{h}_i u_i)}{\sigma_v^2} = -\frac{(u_i - \mu_{*i})^2}{\sigma_{*i}^2} - \frac{\sum \boldsymbol{\varepsilon}_i^2}{\sigma_v^2} + \frac{\frac{\sigma_u^2 \sigma_{u_i}^2 \left(\sum \mathbf{h}_i \boldsymbol{\varepsilon}_i\right)^2}{\left(\sigma_v^2 + \sigma_{u_i}^2 \sum \mathbf{h}_i^2\right)^2}}{\frac{\sigma_v^2 \sigma_{u_i}^2}{\sigma_v^2 + \sigma_{u_i}^2 \sum \mathbf{h}_i^2}}$$

$$= -\frac{(u_i - \mu_{*i})^2}{\sigma_{*i}^2} - \frac{\sum \boldsymbol{\varepsilon}_i^2}{\sigma_v^2} + \frac{\frac{\sigma_u^2 \sigma_{u_i}^2 \left(\sum \mathbf{h}_i \boldsymbol{\varepsilon}_i\right)^2}{\left(\sigma_v^2 + \sigma_{u_i}^2 \sum \mathbf{h}_i^2\right)^2}}{\frac{\sigma_v^2 \sigma_{u_i}^2}{\sigma_v^2 + \sigma_{u_i}^2 \sum \mathbf{h}_i^2}}$$

$$= -\frac{(u_i - \mu_{*i})^2}{\sigma_{*i}^2} - \frac{\sum \boldsymbol{\varepsilon}_i^2}{\sigma_v^2} + \frac{\sigma_{u_i}^2 \left(\sum \mathbf{h}_i \boldsymbol{\varepsilon}_i\right)^2}{\sigma_v^2\left(\sigma_v^2 + \sigma_{u_i}^2 \sum \mathbf{h}_i^2\right)}$$

Define $a_{*i} = \frac{1}{\sigma_v^2}\left(\sum \boldsymbol{\varepsilon}_i^2 - \frac{\sigma_{u_i}^2 (\sum h_i \ \boldsymbol{\varepsilon}_i)^2}{\sigma_v^2 + \sigma_{u_i}^2 \sum \mathbf{h}_i^2}\right) = \frac{1}{\sigma_v^2}\left(\sum \boldsymbol{\varepsilon}_i^2 - \frac{\sigma_{*i}^2}{\sigma_v^2}(\sum h_i \ \boldsymbol{\varepsilon}_i)^2\right)$. Then the term in parentheses in (A.4) becomes:

$$-\frac{u_i^2}{\sigma_{u_i}^2} - \frac{(\boldsymbol{\varepsilon}_i - \mathbf{h}_i u_i)'(\boldsymbol{\varepsilon}_i - \mathbf{h}_i u_i)}{\sigma_v^2} = -\frac{\left(u_i - \mu_{*i}\right)^2}{\sigma_{*i}^2} - a_{*i}$$

The marginal density of $\boldsymbol{\varepsilon}_i$ is obtained by integrating u_i out of the joint density in (A.4):

(A.5) $$f(\boldsymbol{\varepsilon}_i) = \int_0^\infty f(u_i, \boldsymbol{\varepsilon}_i) du_i$$

$$= \int_0^\infty \frac{2}{(2\pi)^{T_i+12} \sigma_v^{T_i} \sigma_{u_i}} \exp\left(-\frac{u_i^2}{2\sigma_{u_i}^2} - \frac{(\boldsymbol{\varepsilon}_i - \mathbf{h}_i u_i)'(\boldsymbol{\varepsilon}_i - \mathbf{h}_i u_i)}{2\sigma_v^2}\right) du_i$$

$$= \int_0^\infty \frac{2\sigma_{*i}}{(2\pi)^{T_i 2} \sigma_v^{T_i} \sigma_{u_i}} \frac{1}{\sqrt{2\pi}\sigma_{*i}} \exp\left(-\frac{(u_i - \mu_{*i})^2}{2\sigma_{*i}^2}\right) \exp\left(-\frac{1}{2} a_{*i}\right) du_i$$

$$= \frac{2\sigma_{*i}}{(2\pi)^{T_i 2} \sigma_v^{T_i} \sigma_{u_i}} \exp\left(-\frac{1}{2} a_{*i}\right) \Phi\left(\frac{u_i - \mu_{*i}}{\sigma_{*i}}\right)\Big|_0^{+\infty}$$

$$= \frac{2\sigma_{*i}}{(2\pi)^{T_i 2} \sigma_v^{T_i} \sigma_{u_i}} \exp\left(-\frac{1}{2} a_{*i}\right) \left(1 - \Phi\left(-\frac{\mu_{*i}}{\sigma_{*i}}\right)\right)$$

$$= \frac{2\sigma_{*i}}{(2\pi)^{T_i 2} \sigma_v^{T_i} \sigma_{u_i}} \exp\left(-\frac{1}{2} a_{*i}\right) \Phi\left(\frac{\mu_{*i}}{\sigma_{*i}}\right)$$

Thus:

(A.6) $$f(\boldsymbol{\varepsilon}_i) = \frac{2\sigma_{*i}}{(2\pi)^{T_i 2}\sigma_v^{T_i}\sigma_{u_i}} \exp\left(-\frac{1}{2}a_{*i}\right)\Phi\left(\frac{\mu_{*i}}{\sigma_{*i}}\right)$$

where:

$$\sigma_{*i}^2 = \frac{\sigma_v^2\sigma_{u_i}^2}{\sigma_v^2+\sigma_{u_i}^2\sum \mathbf{h}_i^2}$$

$$\mu_{*i} = \frac{\sigma_{u_i}^2\sum \mathbf{h}_i\boldsymbol{\varepsilon}_i}{\sigma_v^2+\sigma_{u_i}^2\sum \mathbf{h}_i^2} = \frac{\sigma_{*i}^2}{\sigma_v^2}\sum \mathbf{h}_i\boldsymbol{\varepsilon}_i$$

$$a_{*i} = \frac{1}{\sigma_v^2}\left(\sum \boldsymbol{\varepsilon}_i^2 - \frac{\sigma_{*i}^2}{\sigma_v^2}(\sum \mathbf{h}_i\boldsymbol{\varepsilon}_i)^2\right) = \frac{\sum \boldsymbol{\varepsilon}_i^2}{\sigma_v^2} - \frac{\mu_{*i}^2}{\sigma_{*i}^2}$$

and $\Phi(\cdot)$ is the cdf of the standard normal density. The panel-level likelihood is given by:

(A.7) $$\ln L_i = \ln 2 - \frac{T_i}{2}\ln(2\pi) + \ln\sigma_{*i} - T_i\ln\sigma_v - \ln\sigma_u - \frac{1}{2}\frac{\sum \boldsymbol{\varepsilon}_i^2}{\sigma_v^2} + \frac{1}{2}\frac{\mu_{*i}^2}{\sigma_{*i}^2} + \ln\left(\Phi\left(\frac{\mu_{*i}}{\sigma_{*i}}\right)\right).$$

The log-likelihood for the whole sample $\ln L$ is the sum of the logs of the panel-level likelihoods $\ln L_i$, defined in (A.7):

(A.8) $$\ln L = \sum_{i=1}^{N} \ln L_i$$

To estimate the technical inefficiency, we follow Jondrow et al. (1982) by first deriving the conditional density of u_i given $\boldsymbol{\varepsilon}_i$. For the normal-half-normal case, it is given by:

(A.9) $$f(u_i|\boldsymbol{\varepsilon}_i) = \frac{\exp\left[-\frac{1}{2}\left(\frac{(u_i-\mu_{*i})^2}{\sigma_{*i}^2}\right)\right]}{\sigma_{*i}\sqrt{2\pi}\Phi\left(\frac{\mu_{*i}}{\sigma_{*i}}\right)}$$

and then using the conditional mean of (A.9):

(A.10) $$\hat{E}[u_i|\boldsymbol{\varepsilon}_i] = \mu_{*i} + \sigma_{*i}\frac{\phi\left(\frac{\mu_{*i}}{\sigma_{*i}}\right)}{\Phi\left(\frac{\mu_{*i}}{\sigma_{*i}}\right)}$$

as the inefficiency estimator. Finally, the estimator of the time-varying inefficiency is $\hat{u}_{it} = \mathbf{h}_i E(u_i|\boldsymbol{\varepsilon}_i)$ and the efficiency estimator is $\exp(-\hat{u}_{it})$.

2. The estimation of the model in (8)

First, we write:

(A.11) $$lne_{it} = f(\cdot) + v_{0i} + \varepsilon_{it}$$

where $\varepsilon_{it} = v_{it} + u_{it}$ and $v_{0i} \sim N\left(0, \sigma_{v_0}\right)$. The T_i observations for country i are independent if conditioned on v_{0i} and the conditional density can be written as:

(A.12) $$f\left(\varepsilon_{i1}, \dots, \varepsilon_{iT_i}|v_{0i}\right) = \prod_{i=1}^{T_i} \frac{2\sigma_{*i}}{(2\pi)^{T_i 2}\sigma_v^{T_i}\sigma_{u_i}} \exp\left(-\frac{1}{2}a_{*i}\right)\Phi\left(\frac{\mu_{*i}}{\sigma_{*i}}\right)$$

where σ_{*i}, a_{*i}, and μ_{*i} are defined in the previous section right after the Equation (A.6), where the Model in (4) is derived. Integrating v_{0i} out of (A.12) will yield an unconditional density of $\boldsymbol{\varepsilon}_i$:

(A.13) $$f\left(\varepsilon_{i1}, \dots, \varepsilon_{iT_i}\right) = \int_{-\infty}^{\infty}\left[\prod_{i=1}^{T_i} \frac{2\sigma_{*i}}{(2\pi)^{T_i 2}\sigma_v^{T_i}\sigma_{u_i}} \exp\left(-\frac{1}{2}a_{*i}\right)\Phi\left(\frac{\mu_{*i}}{\sigma_{*i}}\right)\right]\frac{1}{\sigma_{v_0}}\phi\left(\frac{v_{0i}}{\sigma_{v_0}}\right)\mathrm{d}v_{0i}$$

Using (A.13), the panel-level likelihood contribution for country i is then given by:

(A.14) $$\ln L_i(\boldsymbol{\theta}) = \ln \int_{-\infty}^{\infty} \left[\prod_{i=1}^{T_i} \frac{2\sigma_{*i}}{(2\pi)^{T_i 2} \sigma_v^{T_i} \sigma_{u_i}} \exp\left(-\frac{1}{2} a_{*i}\right) \Phi\left(\frac{\mu_{*i}}{\sigma_{*i}}\right) \right] \phi(V_{0i}) \mathrm{d}V_{0i}$$

where $V_{0i} = v_{0i}/\sigma_{v_0} \sim N(0,1)$ and $\boldsymbol{\theta}$ is the set of all parameters to be estimated. The integral in (A.14) does not exist in a closed-form but can be approximated using Monte-Carlo integration. The result is given in the main text as equation (9).

To obtain the inefficiency estimator, it is not enough to use (A.9) and (A.10) because of the term v_{0i} term. v_{0i} needs to be integrated out of:

(A.15) $$E[u_i | \boldsymbol{\varepsilon}_i(v_{0i})] = \mu_{*i}(v_{0i}) + \sigma_{*i}^2 \frac{\phi\left(\frac{\mu_{*i}(v_{0i})}{\sigma_{*i}}\right)}{\Phi\left(\frac{\mu_{*i}(v_{0i})}{\sigma_{*i}}\right)}$$

to get the unconditional estimator:

(A.16) $$\hat{E}[u_i | \text{data}] = \int_{-\infty}^{\infty} \left\{ \mu_{*i}(v_{0i}) + \sigma_{*i} \frac{\phi\left(\frac{\mu_{*i}(v_{0i})}{\sigma_{*i}}\right)}{\Phi\left(\frac{\mu_{*i}(v_{0i})}{\sigma_{*i}}\right)} \right\} f(v_{0i}) dv_{0i}$$

As before, the estimator in (A.16) can be approximated using Monte-Carlo integration. The final formula is given in the main text as equation (11).