Overview of Some Recent Results in Energy Market Modelling and Clean Energy Vision in Canada

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Abstract: The paper overviews our recent results in energy market modelling, including: option pricing formula for a mean-reversion asset; variance and volatility swaps in energy markets; applications of weather derivatives in energy markets; pricing crude oil options using Lévy processes; energy contracts modelling with delayed and jumped volatilities; applications of mean-reverting processes in Alberta energy markets; Aaternatives to Black-76 model for options valuation of futures contracts. We will also consider the clean renewable energy prospective in Canada, and, in particular in Alberta and Calgary.

Keywords: energy markets; option pricing; mean-reverting assets; variance and volatility swaps; weather derivatives; Lévy processes; delayed and jumped volatilities; Alberta energy markets; alternatives to Black-76 model; wind, solar and water energy.

1 A Brief Introduction

In this paper we give overview of some recent results in energy markets modelling, and consider the clean renewable energy prospective in Canada, and, in particular, in Alberta. This brief introduction gives a quick insight into the eight papers in energy markets modelling we published during the period of 2008-2021.

Some commodity prices, like oil and gas, exhibit the mean reversion, unlike stock price. It means that they tend over time to return to some long-term mean. We presented explicit option pricing formula for a mean-reverting asset in energy market in the paper "Explicit Option Pricing Formula for a Mean-Reverting Asset in Energy Market "(*J. Numer. Appl. Math.*, V.1(96), 2008, 216-233).

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We calculate variance and volatility swaps in energy markets in "Variance and Volatility Swaps in Energy Markets" (*The J. Energy Markets*, V. 6, N.1, Spring 2013, 33-50).

We use future contracts written on temperature to demonstrate the hedging strategies for commodities as an application of weather derivatives in "Weather Derivatives in Energy Markets" (*The J. Energy Markets*, V.8, N.1, March 2015, 59-76). Our focus was on the dynamic hedging strategy of energy futures using temperature futures and constructing the hedge ratio.

Crude oil prices exhibit significant volatility over time and the distribution of returns on crude oil prices show at tails and skewness, and they barely follow normal distribution. This is the reason we use normal inverse Gaussian process, jump diffusion process, and variance-gamma process as three Lévy processes that do not have these drawbacks, and their tails carry heavier mass than normal distribution. Our results indicate that all these three Levy processes have very good out of sample results for near at the money options than others. See "Pricing Crude Oil Options using Lévy Processes" (*The J. Energy Markets*, V.9, N 1, March 2016, 47-64).

We also considered stochastic modelling and pricing of energy markets' contracts for stochastic volatilities with delay and jumps. Our model of stochastic volatility exhibits jumps and also past-dependence: the behaviour of a stock price right after a given time t not only depends on the situation at t, but also on the whole past (history) of the process $S(t)$ up to time t. The basic products in these markets are spot, futures and forward contracts and options written on these. We study forwards and swaps. A numerical examples is presented for stochastic volatility with delay using the Henry Hub daily natural gas data (1997-20011). See "Energy Market Contracts with Delayed and Jumped Volatilities" (*Handbook of Energy Finance: Theories, Practices and Simulations*, World Scientific, 2019)

In "Applications of Mean-reverting Processes in Alberta Energy Markets" (*Mathematics* (IF2.258), Pub Date : 2021-03-25) we introduced fuel-switching price, which designed for encouraging power plant companies to switch from coal to natural gas when they produce electricity and successfully applied on European market, to Albertan Market. Moreover, we introduce energy-switching price which consider power switch from natural gas to wind. We modelled these two prices using five mean reverting processes including Regime-switching processes, Levy-driven Ornstein-Uhlenbeck process and Inhomogeneous Geometric Brownian Motion, and estimate them based on multiple procedures such as Maximum likelihood estimation and Expectation-Maximization algorithm. At last, this paper proves previous result applied on Albertan Market that the jump modelling techniques is needed when modelling fuel-switching data. In addition, it not only gives promising conclusion on the necessity of introducing Regime-switching models to the fuel-switching data, but also show that Regime-switching model is better fitted to the data.

In March 2020, the prompt month WTI futures contract settled below

zero for the first time in the contract's history. Many market participants apply the Black-76 model or some variation when calculating the value of the options on this futures contract as a relatively straightforward, parametric valuation method. This calculation model is hard wired into many commodity trading and risk management systems, when the prices go below zero, and traders and risk managers rely on its straightforward and reproducible output. However, Black 76 requires positive underlying market prices. The negative prompt month settlement price caused considerable consternation among energy traders and risk managers. More generally, OTC options are also available on basis or differential prices. These transactions are options on the difference between two published indexes such as NYMEX Henry Hub and AECO (for natural gas) or Cushing WTI and Houston (for crude oil). As such, these instruments frequently have negative underlying market prices. Our task was to propose alternative models to Black-76 to valuate option prices when the underlying future contracts can assume negative values. The paper "Alternatives to Black-76 Model for Options Valuation of Futures Contracts" (Wilmott, 2021, issue 114, pages 40-49), considers some alternatives to Black-76 model to value European options on future contracts in which the underlying market prices can be negative or/and mean reverting. We specifically consider two models, namely Ornstein-Uhlenbeck (OU), for negative prices, and continuous-time GARCH (or inhomogeneous geometric Brownian motion), for positive prices. We then analyze the results and compare them with Black-76, the most commonly used model, when the underlying market prices are positive. Numerical examples are presented using WTI and NYMEX NG data sets.

Finally, we present a vision to transition to 100% wind, water & solar energy in Canada. A group of U.S. civil engineering has calculated that Canada could be completely powered by renewable energy, if we just decide to do it. They say that would save $110.1 billion on health care costs every year and prevent $9,884$ annual air pollution deaths. Their research is available at thesolutionsproject.org.

The paper organized as follows. Option pricing formula for a mean-reversion asset is considered in Section 2. Variance and volatility swaps valuations in energy markets are presented in Section 3. Applications of weather derivatives in energy markets are reviewed in Section 4. Pricing crude oil options using Levy processes is considered in Section 5. Energy contracts modelling with delayed and jumped volatilities is presented in Section 6. Applications of mean-reverting processes in Alberta energy markets are reviewed in Section 7. Alternatives to Black-76 model for options valuation of futures contracts are considered in Section 8. A vision to transition to 100% wind, water and solar energy in Canada is considered in Section 9. Prospectives of wind and solar energy in Alberta are presented in Section 10. Description of energy transition centre in Calgary, AB, Canada, is mentioned in Section 11. Section 12 concludes the paper.

2 Explicit Option Pricing Formula for a Mean-Reverting Asset in Energy Market [1]

Let us consider a risky asset S_t following the mean-reverting stochastic process given by the following stochastic differential equation

$$dS_t = a(L - S_t)dt + \sigma S_t dW_t,$$

where W is a standard Wiener process, $\sigma > 0$ is the volatility, the constant L is called the 'long-term mean' of the process, to which it reverts over time, and $a > 0$ measures the 'strength' of mean reversion.

This mean-reverting model is a one-factor version of the two-factor model made popular in the context of energy modelling by Pilipovic (1997). We call it *continuous-time GARCH* or *inhomogeneous geometric Brownian motion* model.

Using a change of time method we find an explicit solution of this equation and using this solution we are able to find the variance and volatility swaps pricing formula under the physical measure. Then, using the same argument, we find the option pricing formula under risk-neutral measure. Black's model (1976) and Schwartz's model (1997) have become a standard approach to the problem of pricing options on commodities. These models have the advantage of mathematical convenience, in that they give rise to closed-form solutions for some types of options.

$$\begin{aligned} C_T^* &= e^{-(r+a^*)T} S(0) N(y_+) - e^{-rT} K N(y_-) \\ &+ L^* e^{-(r+a^*)T} [(e^{a^*T} - 1) - \textstyle\int_0^{y_0} z F_T^*(dz)], \end{aligned}$$

where

$$y_+ := \sigma\sqrt{T} - y_0 \quad and \quad y_- := -y_0,$$

$$a^* := a + \lambda\sigma, \quad L^* := \frac{aL}{a + \lambda\sigma},$$

y_0 is the solution of the following equation

$$y_0 = \frac{\ln(\frac{K}{S(0)}) + (\frac{\sigma^2}{2} + a^*)T}{\sigma\sqrt{T}} - \frac{\ln(1 + \frac{a^* L^*}{S(0)} \int_0^T e^{a^* s} e^{-\sigma y_0 \sqrt{s} + \frac{\sigma^2 s}{2}} ds)}{\sigma\sqrt{T}},$$

and $F_T^*(dz)$ is the probability distribution $F_T(dz)$, i.e., cdf of r.v.

$$\eta(T) = \frac{4ae^{-aT}}{\sigma^2} e^{-2B_{\frac{\sigma^2 T}{4}}} \int_0^{\sigma^2 T/4} e^{2((2a/\sigma^2+1)+W_s)} ds,$$

but instead of a we have to take $a^* = a + \lambda\sigma$, λ is a market price of risk. We note that this cdf can be estimated and calculated by M. Yor's (1992) result: On some exponential functions of Brownian motion, *Advances in Applied Probability*, Vol. 24, No. 3, 509-531.

Remark: When $L^* = 0$ and $a^* = -r$, then the explicit option pricing formula is the well-known Black-Scholes formula!

Numerical Example: AECO Natural GAS Index (1 May 1998-30 April 1999). We shall calculate the value of a European call option on the price of a daily natural gas contract. To apply our formula for calculating this value we need to calibrate the parameters a, L, σ and λ. These parameters may be obtained from futures prices for the AECO Natural Gas Index for the period 1 May 1998 to 30 April 1999 (see Bos, Ware and Pavlov (2002), p.340). The parameters pertaining to the option are the following:

Price and Option Process Parameters						
T	a	σ	L	λ	r	K
6 months	4.6488	1.5116	2.7264	0.1885	0.05	3

From this table we can calculate the values for a^* and L^* :

$$a^* = a + \lambda\sigma = 4.9337,$$

and

$$L^* = \frac{aL}{a + \lambda\sigma} = 2.5690.$$

For the value of S_0 we can take $S_0 \in [1, 6]$.

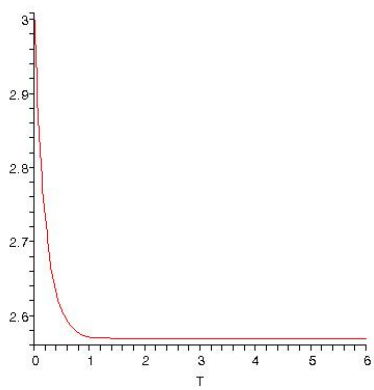

Fig. 1. Dependence of ES_t on T (AECO Natural Gas Index (1 May 1998-30 April 1999))

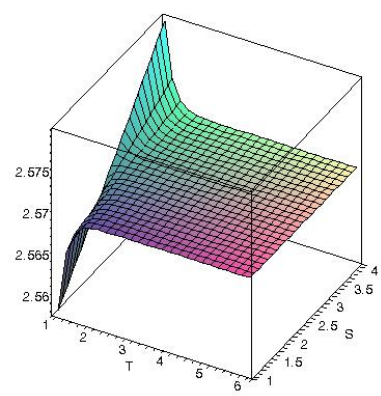

Fig. 2. Dependence of ES_t on S_0 and T (AECO Natural Gas Index (1 May 1998-30 April 1999))

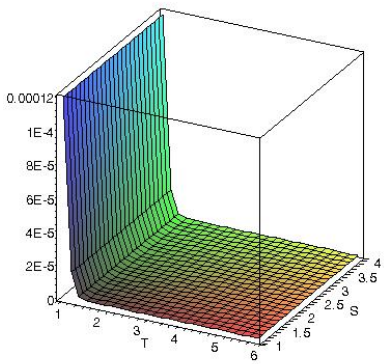

Fig. 3. Dependence of variance of S_t on S_0 and T (AECO Natural Gas Index (1 May 1998-30 April 1999))

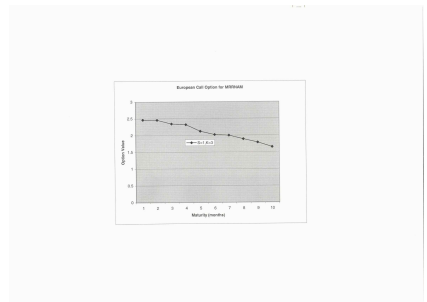

Fig. 4. Dependence of European Call Option Price on Maturity (months) ($S(0) = 1$ and $K = 3$) (AECO Natural Gas Index (1 May 1998-30 April 1999))

3 Variance and Volatility Swaps in Energy Markets [2]

Variance swaps are quite common in commodity, e.g., in energy market, and they are commonly traded. We consider Ornstein-Uhlenbeck process for commodity asset with stochastic volatility following continuous-time GARCH model or Pilipovic (1998) one-factor model. The classical stochastic process for the spot dynamics of commodity prices, as it was mentioned above, is given by the Schwartz' model (1997). It is defined as the exponential of an Ornstein-Uhlenbeck (OU) process, and has become the standard model for energy prices possessing mean-reverting features.

At maturity, a variance swap pays off the difference between the realized variance of the futures contract over the life of the swap and the fixed variance swap rate. And since a variance swap has zero net market value at initiation, absence of arbitrage implies that the fixed variance swap rate equals to conditional risk-neutral expectation of the realized variance over the life of swap. Therefore, e.g., the time-series average of the payoff and/or excess return on a variance swap is a measure of the variance risk premium. Variance risk premia in energy commodities, crude oil and natural gas, has been considered by Trolle and E. Schwartz (2009). The same methodology as in Trolle & Schwartz (2009) was used by Carr & Wu (2009) in their study of equity variance risk premia.

The S & P GSCI is comprised of 24 commodities with the weight of each commodity determined by their relative levels of world production over the past five years. The DJ-AIGCI is comprised of 19 commodities with the weight of each component determined by liquidity and world production values, with liquidity being the dominant factor. Crude oil and natural

gas are the largest components in both indices. In 2007, their weight were 51.30% and 6.71%, respectively, in the S & P GSCI and 13.88%and 11.03%, respectively, in the DJ-AIGCI.

The Chicago Board Options Exchange (CBOE) recently introduced a Crude Oil Volatility Index (ticker symbol OVX). This index also measures the conditional risk-neutral expectation of crude oil variance, but is computed from a cross-section of listed options on the United States Oil Fund (USO), which tracks the price of WTI as closely as possible. The CBOE Crude Oil ETF Volatility Index (Oil VIX, Ticker - OVX) measures the market's expectation of 30-day volatility of crude oil prices by applying the VIX methodology to United States Oil Fund, LP (Ticker - USO) options spanning a wide range of strike prices (see Figures below. Courtesy-CBOE: http://www.cboe.com/micro/oilvix/introduction.aspx).
We have to notice that crude oil and natural gas trade in units of 1,000 barrels and 10,000 British thermal units (mmBtu), respectively. Usually, prices are quoted as US dollars and cents per barrel or mmBtu.

We consider a risky asset in energy market with stochastic variance following a mean-reverting stochastic process satisfying the following SDE (continuous-time GARCH(1,1) model):

$$d\sigma^2(t) = a(L - \sigma^2(t))dt + \gamma\sigma^2(t)dW_t,$$

where a is a speed of mean reversion, L is the mean reverting level (or equilibrium level), γ is the volatility of volatility $\sigma(t)$, W_t is a standard Wiener process. Using a change of time method we find an explicit solution of this equation, and using this solution we are able to find the variance and volatility swaps pricing formula under the physical measure. Then, using the same argument, we find the option pricing formula under risk-neutral measure. We applied Brockhaus-Long (2000) approximation to find the value of volatility swap. A numerical example for the AECO Natural Gas Index for the period 1 May 1998 to 30 April 1999 is presented.

Risk-neutral stochastic volatility model has the following form:

$$d\sigma^2(t) = a^*(L^* - \sigma^2(t))dt + \gamma\sigma^2(t)dW_t^*,$$

where

$$a^* := a + \lambda\gamma, \quad L^* := \frac{aL}{a + \lambda\gamma},$$

$W_t^* := W_t + \lambda t$, and λ is the market price of risk.

For the variance swap we have:

$$E^*\sigma_R^2 := EV := \frac{1}{T}\int_0^T E\sigma^2(t)dt = \frac{(\sigma^2(0) - L^*)}{a^*T}(1 - e^{-a^*T}) + L^*.$$

For the volatility swap we obtain (convexity adjustment formula):

$$E^*\sqrt{V} \approx \sqrt{E^*V} - \frac{Var^*(V)}{8(E^*V)^{3/2}}.$$

Numerical Example (AECO Natural Gas Index for the period 1 May 1998 to 30 April 1999):

Parameters			
a	γ	L	λ
4.6488	1.5116	2.7264	0.18

From this table we can calculate the values for risk adjusted parameters a^* and L^* :

$$a^* = a + \lambda\gamma = 4.9337,$$

and

$$L^* = \frac{aL}{a + \lambda\gamma} = 2.5690.$$

For the value of $\sigma^2(0)$ we can take $\sigma^2(0) = 2.25$.

For variance swap and for volatility swap with risk adjusted parameters we use formula obtained above.

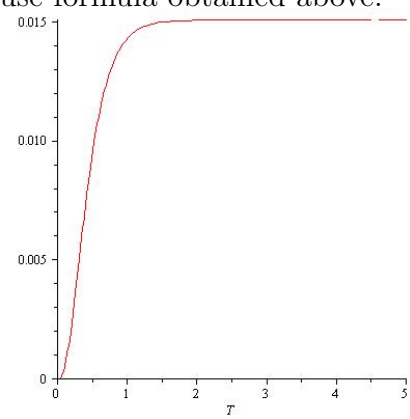

Fig. 1: Variance Swap

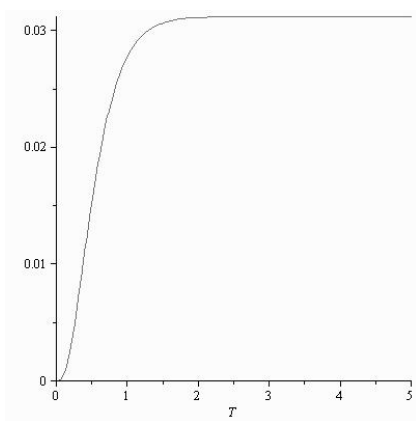

Fig. 2: Volatility Swap

Variance and Volatility Swaps in Energy Markets: Figures
(*The J. Energy Markets, V. 6, N.1, Spring 2013, 33-50*)

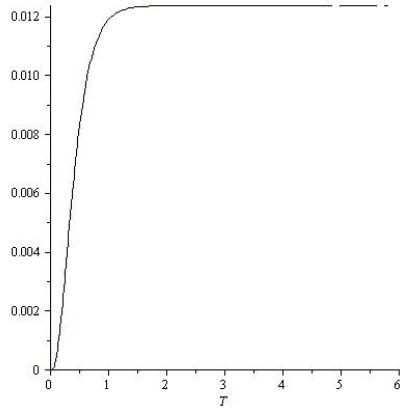

Fig. 3: Variance Swap (Risk Adjusted Parameters)

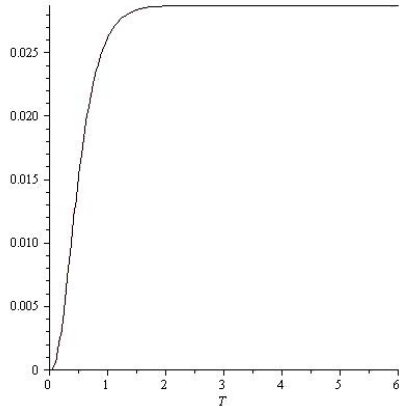

Fig. 4: Volatility Swap (Risk Adjusted Parameters)

Variance and Volatility Swaps in Energy Markets: Figures
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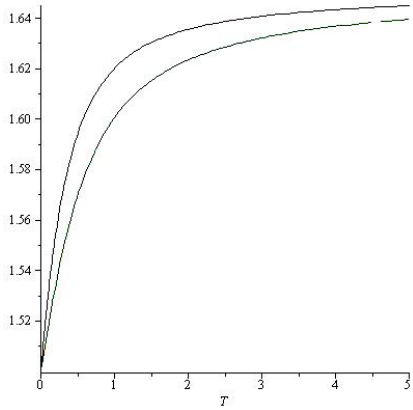


Fig. 5: Comparison: Adjusted and Non-Adjusted Price

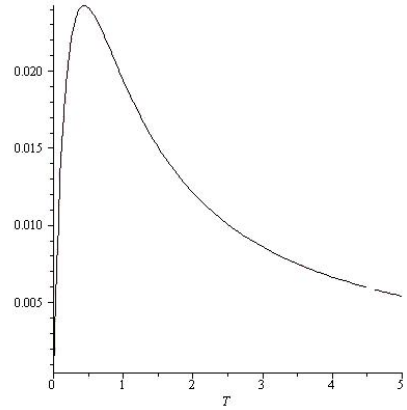


Fig. 6: Convexity Adjustment

4 Weather Derivatives in Energy Markets [3,4]

The weather derivatives market, in which contracts written on weather indices was firstly appeared over-the-counter (OTC) in July 1996 between Aquila Energy and Consolidated Edison Co. from United States. After that, companies accustomed to trading weather contracts based on electricity and gas prices in order to hedge their price risks realized by weather during the end of 1990s and the beginning of 2000s. Consequently, the market grew rapidly and expanded to other industries and to Europe and Japan.

Reported from Weather Risk Management Association (WRMA), an industry body that represents the weather market, recently, the total notional value of the global weather risk market has reached \$11.8 billion in last year. With geographic expansion, the OTC market boosted nearly 30% in last year. In this article, we will concentrate on the market of temperature derivatives found at the Chicago Mercantile Exchange (CME), which is one of the largest weather derivatives trading platforms. Up to now, the CME has weather futures and options traded based on a range of weather indices for 47 cities from United States, Canada, Europe, Australia and Asia.

As a common sense, weather affects different entities in different ways. In order to hedge these different types of risks, weather derivatives are written on different types of weather variables or weather indices. The most commonly used weather variable is the temperature. Widely used temperature indices include cumulative average temperature (CAT), heating degree days (HDD) and cooling degree days (CDD). They are originated from the energy industry, and designed to correlate well with the local demands for heating or cooling.

CAT is defined as the sum of the daily average temperature over the period $[\tau_1, \tau_2]$ of the contract, the index CAT:$=\sum_{t=\tau_1}^{\tau_2} T(t) = \int_{\tau_1}^{\tau_2} T(t)dt$, where

$T(t)$ is the daily average temperature. It is mainly used in Europe and Canada. In winter, HDD are used to measure the demand for heating, i.e. they are a measure of how cold the weather is and usually used in United States, Europe, Canada and Australia. In contrast, CDD are used in summer to measure the demand of energy used for cooling and a measure of how hot the weather is. They are usually used in United States, Canada and Australia.

The definitions for HDD and CDD are given by HDD:=max$(T(t) - c, 0)$ and CDD:=max$(c - T(t), 0)$, where the constant c denotes the threshold, say $65°F(18°C)$. Since most air conditioners are switched on when temperatures are above or below c.

With respect to our model, consider the weather index $T(t)$, which is the daily average temperature (DAT). We suppose the DAT has a generalization of the Ornstein-Uhlenbeck dynamics

$$dT(t) = ds(t) + k(T(t) - s(t))dt + \sigma(t)dL(t),$$

where $L(t)$ is a Lévy process (jump-diffusion), $s(t)$ is the seasonal mean level and k is the speed in which the temperature reverts to $s(t)$. $\sigma(t)$ is assumed to be a measurable and bounded function represents the seasonal volatility of temperature. In the simplest case, $L(t) = W(t)$-a standard Wiener process.

This model was firstly introduced by Dornier and Queruel (2000) with Brownian motion as the random noise. Benth and Saltyte-Benth (2005) has successfully applied this model with generalized hyperbolic Lévy process to the Norwegian temperature data. We applied this model to our Canadian temperature data (Swishchuk & Cui (2013)).

We define the temperature futures prices written on CAT, CDD and HDD, which constitute the three main classes of futures products at CME market. Consider the price dynamic of future written on CAT over specific time period $[\tau_1, \tau_2]$, with $\tau_1 < \tau_2$. Firstly, assume the daily average temperature follows stochastic differential equation with $L(t)$ being Lévy process and a constant continuously compounding interest rate r.

The future price $F_{CAT}(t, \tau_1, \tau_2)$ at time $0 \leq t \leq \tau_1$ based on CAT under risk-neutral probability measure Q is:

$$F_{CAT}(t, \tau_1, \tau_2) = E^Q[\int_{\tau_1}^{\tau_2} T(s)ds|\mathcal{F}_t],$$

where Q is the risk-neutral measure (specified through Esscher transform) and $\mathcal{F}_t$ is σ-algebra generated by $L(t)$.

Similarly, the risk-neutral CDD and HDD future prices are defined as:

$$F_{CDD}(t, \tau_1, \tau_2) = E^Q[\int_{\tau_1}^{\tau_2} \max(T(s) - c, 0)ds|\mathcal{F}_t],$$

and

$$F_{HDD}(t, \tau_1, \tau_2) = E^Q[\int_{\tau_1}^{\tau_2} \max(c - T(t), 0)ds|\mathcal{F}_t],$$

The relationship between futures prices of CAT, CDD and HDD is defined as

$$F_{CAT}(t, \tau_1, \tau_2) + F_{HDD}(t, \tau_1, \tau_2) = c(\tau_2 - \tau_1) - F_{CDD}(t, \tau_1, \tau_2).$$

We use future contracts written on temperature to demonstrate the hedging strategies for commodities as an application of weather derivative.

Within several forms of weather derivatives, the future contract does not require cost to enter a position, since when entering a future contract, the probability of weather event being lower or higher than the threshold is the same to both side, either side has the same chance of receiving payoff from the counter party.

There are two types of hedging strategies using temperature futures in the following contents.: the first strategy is a a) static hedging mainly focusing on mitigating the volume risk of commodity sales using temperature futures; the other strategy consider the b) dynamic hedging strategy of commodity future using temperature futures:

a) In a static hedge, the number of hedging contracts is not changed over the course of the hedge in response to any movement in the values of the hedging instrument or the hedged asset.

b) In a dynamic hedge, on the other hand, more hedging contracts are bought or sold to bring back the hedge ratio to the target hedge ratio.

A hedge ratio is the ratio of exposure to a hedging instrument to the value of the hedged asset. A ratio of 1 or 100% means that the position is fully hedged and a ratio of 0 means it is not hedged at all.

Without loss of generality, we choose the energy market as the one to hedge using temperature futures.

Therefore, our focus will be on the dynamic hedging strategy of energy futures using temperature futures. In the spirit of Broadie and Jain (2008), consider a portfolio at time t containing one unit of energy (e.g. heating oil) future F_E and β_t (β_t is the hedge ratio for energy future F_E) units of weather futures F_W, both with maturity (delivery) at time T. Assume the portfolio has value $\Pi(t)$ at time t, a constant risk-free interest rate r, then

$$\Pi(t) = e^{-r(T-t)}[F_E(t) + \beta_t F_W(t)]. \tag{1}$$

The portfolio is self-financing, so the change in this portfolio in a small amount of time dt is given by

$$d\Pi(t) = r\Pi(t)dt + e^{-r(T-t)}[dF_E(t) + \beta_t dF_W(t)]. \tag{2}$$

Hence, in order to dynamically hedge the energy future F_E with maturity T, the stochastic component of portfolio vanishes, the hedge ratio β_t could be defined as

$$\beta_t = -\frac{dF_E(t)}{dF_W(t)}, \tag{3}$$

with an assumption that $dF_W(t) \neq 0$. Therefore, from the last equation, to hedge an energy futures, we are required to hold β_t units of temperature future at time t.

Therefore, we need to specify two models for energy and temperature futures so that we could get the explicit dynamics of energy and temperature futures, and hence get a closed form of the hedge ratio β_t. For futures pricing purpose, these models will be built on the underlings of futures, namely the energy spot price and the daily average temperature.

Our energy and temperature models under risk-neutral measure Q are:

$$dX(t) = \left(\theta\sigma_E + \kappa_E\left(\mu - \frac{\sigma_E^2}{2\kappa_E} - X(t)\right)\right)dt + \sigma_E dW_E^\theta(t),$$

and

$$dT(t) = ds(t) + (\theta\sigma_W(t) + \kappa_W(T(t) - s(t)))dt + \sigma_W(t)dW_T^\theta(t),$$

where θ is the market price of risk, $X(t) = \ln S(t)$, $W_E^\theta(t)$ and $W_T^\theta(t)$ are Brownian motions (with correlation ρ) w.r.t. Q.

Combined Q dynamics system for energy futures F_E and CAT futures F_W is

$$\begin{cases} dF_E(t,T) &= \sigma_E e^{-\kappa_E(T-t)} \exp\left(\mu_X(T) + \frac{1}{2}\sigma_X^2(T)\right) dW_E^\theta(t); \\ dF_W(t,\tau_1,\tau_2) &= \kappa_W^{-1}(e^{\kappa_W(\tau_2-t)} - e^{\kappa_W(\tau_1-t)})\sigma_W(t)dW_W^\theta(t); \\ dW_E^\theta(t)dW_W^\theta(t) &= \rho dt, \end{cases}$$

where $\mu_X(T)$ and $\sigma_X^2(T)$ are expectation and variance of the log-spot price $X(t)$. Therefore,

$$\beta_t = -\frac{c_1(t)}{c_2(t)}\rho,$$

where $c_1(t)$ and $c_2(t)$ are time-dependent constant defined as:

$$\begin{cases} c_1(t) &:= \sigma_E e^{-\kappa_E(T-t)} \exp\left(\mu_X(T) + \frac{1}{2}\sigma_X^2(T)\right); \\ c_2(t) &:= \kappa_W^{-1}(e^{\kappa_W(\tau_2-t)} - e^{\kappa_W(\tau_1-t)})\sigma_W(t). \end{cases}$$

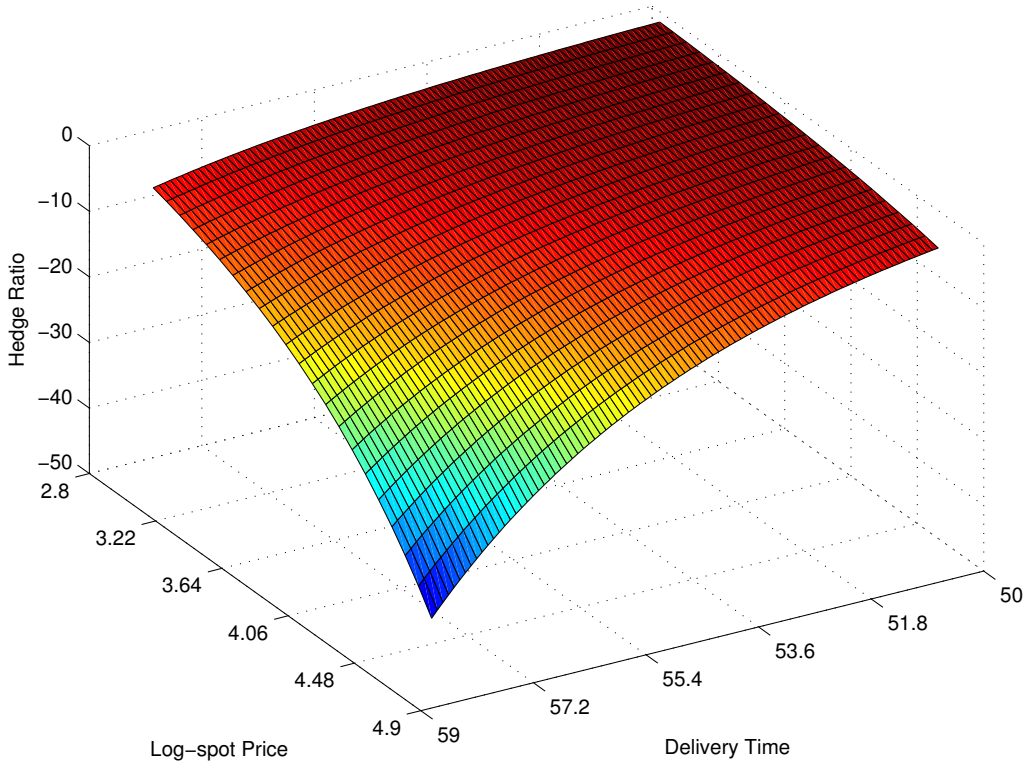
We choose the crude oil (crude oil is the world's most actively traded commodity, and the NYMEX (CME) division light, sweet crude oil futures contract is the world's most liquid forum for crude oil trading) futures as the the one that we want to hedge using CAT futures. Followed by the calibration method described in Schwarz (1997), the log-future prices $\ln F_E(t,T)$ need to be rewritten as the standard state-space form and then applied to Kalman filter to get the parameter set $\Theta_E = \{\kappa_E, \mu_E, \sigma_E, \theta_E\}$ and spot price series $S(t)$.

The data used to calibrate the energy future consist of daily generic observations of WTI light, sweet crude oil futures prices (these data are obtained from Bloomberg financial service) with delivery periods in the first two front months. The WTI crude oil futures data used in calibration cover the CME exchange daily settlement prices ranging from January 2nd, 2001 to December 31st, 2010, resulting in 2508 record for each future contracts set (this choice of data set is consistent with that in Swishchuk and Cui (2013), which is 10 years of temperature data from January 1st, 2001 to December 31st, 2010 in Calgary, AB, Canada). Since there is no exact delivery date for each contract, instead, the CME contract specification defines a delivery period ranging from the first calendar day to the last calendar day of the delivery month, we simply assume that the delivery date for each contract is the first calendar day in the delivery month to calculate the time to maturity value $T_i - t$.

Table below presents the estimation results for the energy model applied to the WTI crude oil future price data. The last two parameters ξ_1 and ξ_2 are the diagonal entries of matrix $H := Var(\epsilon_t)$ with random noise ϵ_t.

Parameter	μ	σ_E	κ_E	θ	ξ_1	ξ_2
Estimation	3.9187	0.0215	0.0025	0.2009	0.0003	0.0123

For the temperature market, we follow the calibration procedure described in Swishchuk and Cui (2013) to get the parameter set $\Theta_W = \{\kappa_W, \sigma_W\}$. For illustration purpose, we choose the estimated parameters in Calgary as the ones under the temperature market to calculate the hedge ratio. Recall the calibration results for Calgary in Swishchuk and Cui (2013), we could get the parameter set $\Theta_W = \{\kappa_W, \sigma_W\}$ in Calgary as follows:
$\kappa_W = -0.2411$
and annual seasonal volatility $\sigma_W = 4.424 + 1.633\cos(0.0167t) + 0.1912\sin(0.0167t)$.
To calculate the correlation parameter ρ, we use the correlation between the filtered log-spot price and daily average temperature as a natural approximation to ρ. By taking all the daily average temperature on the dates with future prices available, and calculating the correlation coefficient between log-spot prices and average temperature of these days over 10 years (from January 2nd, 2001 to December 31st, 2010), we have the correlation $\rho = 0.1058$. This correlation indicates a positive correlation between the log-spot price of crude oil and daily average temperature. With the calibrated parameters in energy model and temperature model, we could then calculate the dynamic hedge ratio β_t explicitly. In the Figure below, we plot the initial hedge ratio β_0 along the crude oil future delivery time (in days) and initial log-spot price dimensions.



From this Figure, we could find that if one hold a crude oil futures, initially he need to short some CAT futures in the portfolio depending on the spot price of the crude oil and the time to delivery (trade termination) length. Basically the number of temperature futures one need to hold will be more with longer time to delivery and higher spot price of the crude oil. Moreover, we could conclude that the same effect holds for other energy commodities, such as heating oil, gas and so on, since they are usually positively correlated to the crude oil market movement.

5 Pricing Crude Oil Options using Lévy Processes [5]

Crude oil prices exhibit significant volatility over time and the distribution of returns on crude oil prices show fat tails and skewness, and they barely follow normal distribution. This is the reason we use Normal Gaussian Process (NIG), Jump Diffusion Process (JD), and Variance-Gamma Process (VG) as three Levy processes that do not have these drawbacks and their tails carry heavier mass than normal distribution. We use fractional fast Fourier transform to calibrate parameters in an optimization setup, using data on European-style options on crude oil futures in NYMEX for the settlement

date of April 24th, 2015. Our results indicate that all these three Levy processes have very good out of sample results for near at the money options than others.

We consider:
Merton's (1976) Jump Diffusion Model:

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t + (e^{\alpha+\beta\epsilon} - 1)dN_t,$$

where Brownian motion W_t and Poisson process N_t are independent, $\epsilon \approx N(0,1)$;

Normal Inverse Gaussian (NIG) Model:

$$S_t = S_0 \exp\{\mu^Q t + X_t\},$$

where μ^Q is the drift under Q measure,

$$X_t = \beta\delta^2 I_t + \delta W_{I_t}$$

is a NIG process, I_t is the inverse Gaussian process. NIG process has three parameters, tail-heaviness α, skewness β and scale δ;

Variance Gamma (VG) Model:

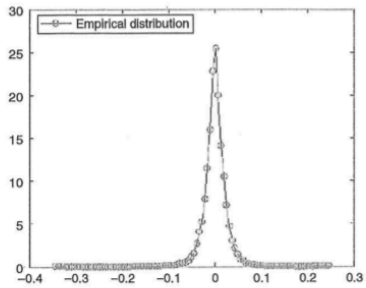
$$S_t = S_0 \exp\{\mu^Q t + X_t\},$$

where X_t is a VG process such that

$$X_t = \theta I_t + \sigma W_{I_t},$$

and I_t is a gamma process with parameter v.
Figures, Tables and Estimations are shown below.

FIGURE 1 Empirical distribution of returns on WTI spot crude oil prices from January 4, 1983 to April 21, 2015.



Source: Bloomberg.

TABLE 1 Descriptive statistics of returns on WTI spot crude oil from 1990 Q1 through 2015 Q4.

Period	Observation	Mean	SD	Skewness	Kurtosis
January 1, 1990–December 31, 1992	750	0.0003	0.031	−1.6856	28.3106
January 1, 1993–December 31, 1996	753	0.0001	0.016	−0.4139	5.9106
January 1, 1997–December 31, 1999	503	−0.0012	0.0262	0.9451	11.6558
January 1, 2000–December 31, 2002	747	0.0006	0.0265	−0.4389	5.5271
January 1, 2003–December 31, 2005	750	0.0012	0.0241	−0.294	4.3453
January 1, 2006–December 31, 2008	749	0	0.0279	1.0834	14.507
January 1, 2009–December 31, 2012	753	0.0014	0.0258	0.1359	6.7841
January 1, 2013–December 31, 2015	826	−0.0006	0.0167	0.0406	5.7803

Source: Bloomberg. SD denotes standard deviation.

Fig. 1. Dependence of ES_t on T (AECO Natural Gas Index (1 May 1998-30 April 1999))

Fig. 2. Dependence of ES_t on S_0 and T (AECO Natural Gas Index (1 May 1998-30 April 1999))

TABLE 3 WTI crude futures and options prices with strikes.

	Days to expiry after market close on April 24, 2015						
	18	52	113	205	297	387	417
	Settlement of each contract						
Strike	June 2015	July 2015	Sept 2015	Dec 2015	March 2016	June 2016	July 2016
WTI crude futures options							
50.00	7.36	9.60	11.78	13.86	14.94	15.61	15.61
51.00	6.43	8.73	10.96	13.06	14.31	15.04	15.04
52.00	5.53	7.89	10.15	12.28	13.38	14.29	14.29
53.00	4.68	7.08	9.37	11.52	12.80	13.55	13.55
54.00	3.87	6.30	8.62	10.78	11.88	12.83	12.83
55.00	3.11	5.54	7.89	10.07	11.16	11.88	11.88
56.00	2.44	4.84	7.19	9.37	10.65	11.19	11.19
57.00	1.86	4.19	6.52	8.70	9.77	10.52	10.52
58.00	1.35	3.58	5.88	8.05	9.11	9.87	9.87
59.00	0.96	3.01	5.27	7.42	8.48	9.24	9.24
60.00	0.67	2.51	4.69	6.81	7.87	8.63	8.63
61.00	0.45	2.07	4.15	6.23	7.29	8.05	8.05
62.00	0.31	1.67	3.66	5.67	6.73	7.48	7.48
63.00	0.21	1.35	3.21	5.16	6.19	6.94	6.94
64.00	0.15	1.08	2.79	4.67	6.00	6.43	6.43
65.00	0.11	0.85	2.40	4.22	5.24	5.95	5.95
66.00	0.09	0.68	2.07	3.79	5.10	5.80	5.80
67.00	0.07	0.53	1.78	3.40	4.39	5.05	5.05
68.00	0.06	0.42	1.53	3.05	4.00	4.64	4.64
Futures							
	57.15	58.90	60.50	62.03	62.98	63.57	63.68

Fig. 3. Dependence of variance of S_t on S_0 and T (AECO Natural Gas Index (1 May 1998-30 April 1999))

TABLE 5 Calibrated parameters of JDM, VG and NIG processes.

Parameters	JDM	VG	NIG
θ	N/A	0.1820	N/A
δ	0.4628	N/A	2.5327
α	0.1247	N/A	−18.5203
λ	0.3318	N/A	N/A
σ	0.4272	0.4104	N/A
ν	N/A	0.0823	N/A
β	N/A	N/A	−7.4572

Fig. 4. Dependence of European Call Option Price on Maturity (months) ($S(0) = 1$ and $K = 3$) (AECO Natural Gas Index (1 May 1998-30 April 1999))

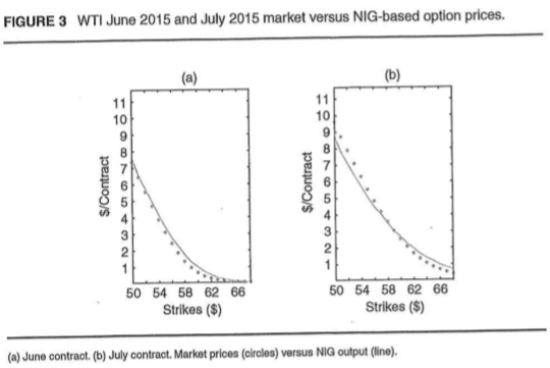


Fig. 1. Dependence of ES_t on T (AECO Natural Gas Index (1 May 1998-30 April 1999))

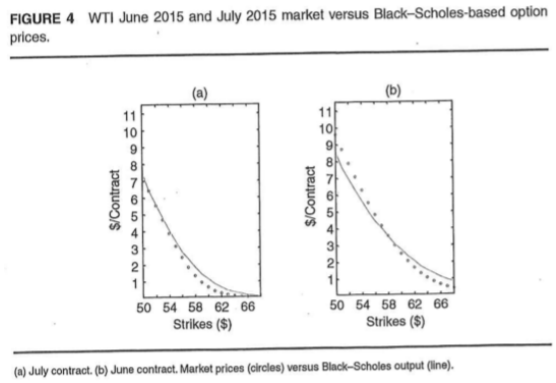


Fig. 2. Dependence of ES_t on S_0 and T (AECO Natural Gas Index (1 May 1998-30 April 1999))

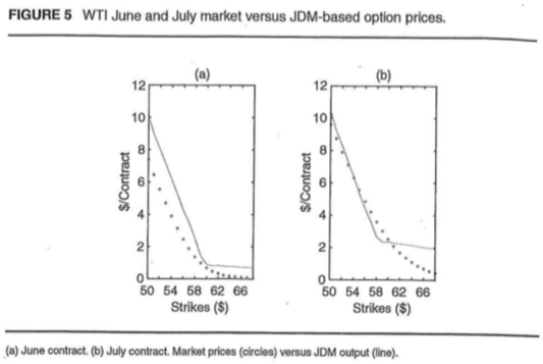


Fig. 3. Dependence of variance of S_t on S_0 and T (AECO Natural Gas Index (1 May 1998-30 April 1999))

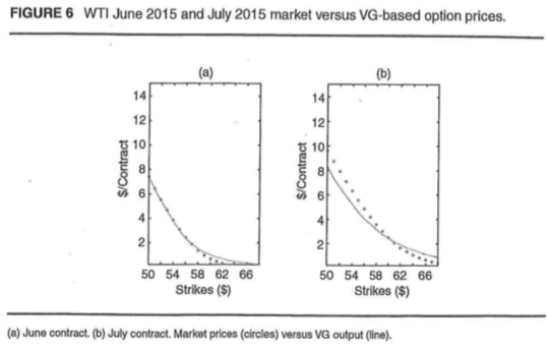
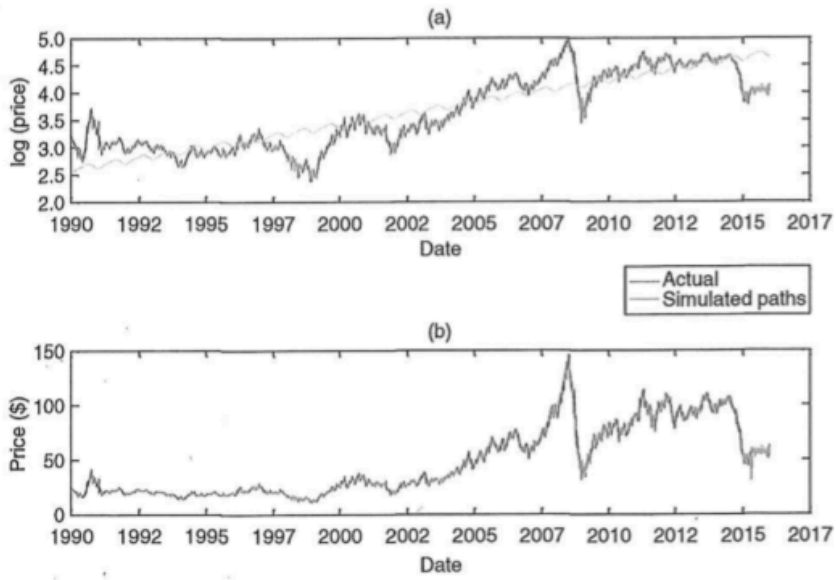


Fig. 4. Dependence of European Call Option Price on Maturity (months) ($S(0) = 1$ and $K = 3$) (AECO Natural Gas Index (1 May 1998-30 April 1999))

FIGURE 7 Log and level of WTI crude spot prices with simulated paths using option-based calibrated the MJDM.



(a) Actual log of WTI crude spot and the simulated path as of April 27, 2015. (b) Actual WTI crude spot and the simulated price path as of April 27, 2015.

Fig. 4. Dependence of European Call Option Price on Maturity (months) ($S(0) = 1$ and $K = 3$) (AECO Natural Gas Index (1 May 1998-30 April 1999))

The volatility of crude oil prices is very important for policy makers, crude oil producers and refineries. We used most recent data through April 2016 from crude oil futures and options markets to model dynamics of crude oil prices. Our results indicate that crude oil prices show significant jumps that are very frequent. Crude oil price returns show skew as well. These findings are consistent across all three models we used in this research.

In the case of JDM, the volatility of size of the jumps is bigger than volatility of the diffusion part. The VG process results in slightly smaller

volatility than JDM. The mean of the jump component size implied by JDM, and skew parameter of VG process both indicate existence of right-skew in crude oil price returns, but the NIG process implies that the density of returns are skewed to the left.

6 Energy Market Contracts with Delayed and Jumped Volatilities [6]

We consider in this section stochastic modelling and pricing of energy markets' contracts for stochastic volatilities with delay and jumps. Our model of stochastic volatility exhibits jumps and also past-dependence: the behaviour of a stock price right after a given time t not only depends on the situation at t, but also on the whole past (history) of the process $S(t)$ up to time t. The spot price process $S(t)$ is modelled by the OU process driven by independent increments process. The basic products in these markets are spot, futures and forward contracts and options written on these. We study forwards and swaps. A numerical examples is presented for stochastic volatility with delay using the Henry Hub daily natural gas data (1997-20011). **Definition of IIP** (see [Skorokhod, 1991], [Benth *et al.*, 2008]): An adapted RCLL stochastic process $I(t)$ starting at zero is an *IIP* (Independent Increment proces) if it satisfies the following two conditions:

1) The increments $I(t_0), I(t_1) - I(t_0), ..., I(t_n) - I(t_{n-1})$ are independent r.v. for any partition $0 \leq t_0 < t_1 < ... < t_n$, and $n \geq 1$.

2) It is continuous in probability, that is, for every $t \geq 0$ and $\epsilon > 0$, it

$$\lim_{s \to t} P(|I(s) - I(t)| > \epsilon) = 0.$$

If we add the condition that increments are stationary, then $I(t)$ is called a Lévy process. (See [Sato, 1999], [Schoutens, 2003]). If the increments of Lévy process are normally distributed then we have a Brownian motion. Lévy processes which are increasing, that is, having only positive jumps, are often called subordinators. Sometimes the IIPs are called additive processes. (See [Sato, 1999]).

Let the stochastic process $S(t)$ be denoted as (we call it Geometric Models with Stochastic Delayed and Jumped Volatility):

$$\ln S(t) = \ln \Lambda(t) + \sum_{i=1}^{m} X_i(t) + \sum_{j=1}^{n} Y_j(t),$$

where for $i = 1, ..., m$

$$dX_i(t) = (\mu_i(t) - \alpha_i(t) X_i(t))dt + \sigma_i(t, X_i(t+\theta))dB(t),$$

and for $j = 1, ..., n$

$$dY_j(t) = (\delta_j(t) - \beta_j(t) Y_j(t))dt + \eta_j(t, Y_j(t+\theta))dI_j(t).$$

Here, $\theta \in [-\tau, 0], \tau > 0$, is the delay, and on the interval $[-\tau, 0]$, $X_i(t) = \phi_i(t)$ and $Y_j(t) = \psi_j(t)$, where $\phi_i(t)$ and $\psi_j(t)$ are deterministic functions, $i = 1, ..., m$ and $j = 1, ..., n$.

We remark that two factors $X_i(t), i = 1, ..., m$, and $Y_j(t), j = 1, ..., n$, represent the long- and short-term fluctuations of the spot dynamics which may be correlated. We suppose that jumps components I_j are independent, which is an obvious restriction of generality.

The deterministic seasonal price level is modelled by the function $\Lambda(t)$, (seasonal function) which is assumed to be continuously differentiable. The coefficients $\mu_i, \alpha_i, \delta_j \beta_j$ are all continuous functions. We suppose that volatilities $\sigma_{ik}(t)$ and $\eta_j(t)$ are stochastic volatilities with delay and jumps. We consider two cases in this situation:

$$\begin{array}{rcl} \frac{d\sigma_i^2(t, X_i(t+\theta))}{dt} & = & \gamma_i^1 V_i + \frac{\alpha}{\tau}[\int_{t-\tau}^t \sigma_i(u, X_i(u+\theta) dB(u) \\ & + & \int_{t-\tau}^t \sigma_i(u, X_i(u+\theta) d\tilde{N}_1(t)]^2 \\ & - & (a_i + b_i)\sigma_i^2(t, X_i(t+\theta)) \end{array}$$

and

$$\begin{array}{rcl} \frac{d\eta_j^2(t, Y_j(t+\theta))}{dt} & = & \gamma_j^2 W_i + \frac{\alpha}{\tau}[\int_{t-\tau}^t \eta_j(u, X_j(u+\theta) dB_1(u) \\ & + & \int_{t-\tau}^t \sigma_i(u, X_i(u+\theta) d\tilde{N}_2(t)]^2 \\ & - & (c_j + d_j)\eta_j^2(t, X_i(t+\theta) \end{array}$$

Here, $B(t)$ and $B_1(t)$ are two independent Brownian motions and $\tilde{N}_1(t)$ and $\tilde{N}_2(t)$ are two independent compensated Poisson processes with intensities λ_1 and λ_2, independent of $B(t)$ and $B_1(t)$.

We note, that in [Benth *et al.*, (2008)] it was considered only deterministic $\sigma_i(t)$ and $\eta_j(t)$.

Let the stochastic process $S(t)$ be defined as (we call it Arithmetic Models with Stochastic Delayed and Jumped Volatility)

$$S(t) = \Lambda(t) + \sum_{i=1}^m X_i(t) + \sum_{j=1}^n Y_j(t),$$

where $X_i(t), i = 1, ..., m$, and $Y_j(t), j = 1, ..., n$, are defined for the geometric models above and the seasonality function $\Lambda(t)$ is the same.

We suppose that for this model the volatilities $\sigma_i^2(t, X_i(t+\theta))$ and $\eta_j^2(t, Y_j(t+\theta))$ satisfied the same equations as for the case of Geometric Models.

We study the pricing of forwards and swaps for the above-mentioned model with delayed and jumped volatilities.

When entering the forward contract, one agrees on a future delivery time and the price to be paid for receiving the underlying. Suppose that the delivery time is T, with $0 \leq t \leq T < +\infty$, and that the agreed price to pay upon delivery is $f(t, T)$:

$$f(t, T) = E_Q[S(T)|\mathcal{F}_t] -$$

fundamental pricing relation between the spot and forward price. Since the energy markets are incomplete, the choice of martingale measure Q is open.

Let us consider swaps, using the electricity market as the typical example. The buyer of an electricity futures receives power during a settlement period (physically or financially), against paying a fixed price per MWh. Let $F(t, \tau_1, \tau_2)$ be the electricity futures price at time t for the delivery period $[\tau_1, \tau_2]$ with $\tau_1 \leq \tau_2$.

In general, we can write the link between a swap contract and the underlying spot as

$$F(t, \tau_1, \tau_2) = E_Q[\int_{\tau_1}^{\tau_2} w(u, \tau_1, \tau_2) S(u) du | \mathcal{F}_t],$$

where w is a weight function.

The dynamics of forward price, $t \to f(t, T)$, wrt Q^θ in the Geometric Model case is

$$\begin{aligned} \frac{df(t,T)}{f(t,T)} &= \{\sum_{i=1}^m \sigma_i(t, X_i(t+\theta)) \exp(-\int_t^T \alpha_i(u) du)\} dB^\theta(t) \\ &+ \sum_{j=1}^n \{\int_R \exp(z\eta_j(t, Y_j(t+\theta)) e^{-\int_t^\tau \beta_j(u) du}) - 1\} \tilde{N}_j^\theta(dt, dz). \end{aligned}$$

The risk-neutral dynamics of the swap price $F(t, \tau_1, \tau_2)$ in the Geometric Models case is given by

$$\begin{aligned} \frac{dF(t,\tau_1,\tau_2)}{F(t-,\tau_1,\tau_2)} &= \sum_{i=1}^m \sigma_i(t, X_i(t+\theta)) dB^\theta(t) \\ &+ \sum_{j=1}^n \int_R (e^{\eta_j(t, Y_j(t+\theta))z} - 1) \tilde{N}_j^\theta(dz, dt). \end{aligned}$$

The risk-neutral dynamics of the swap price $F(t, \tau_1, \tau_2)$ in the Arithmetic Models case is given by

$$\begin{aligned} dF(t, \tau_1, \tau_2) &= \sum_{i=1}^m \sigma_i(t, X_i(t+\theta)) \int_{\tau_1)}^{\tau_2} w(u, \tau_1, \tau_2) e^{-\int_v^u \alpha_i(s) ds} du dB^\theta(t) \\ &+ \sum_{j=1}^n \int_R z\eta_j(t, Y_j(t+\theta)) \\ &\times \int_{\tau_1}^{\tau_2} w(u, \tau_1, \tau_2) e^{-\int_v^u \beta_j(s) ds} du \tilde{N}_j^\theta(dt, dz). \end{aligned}$$

Numerical Example: Henry Hub Natural Gas Daily Spot Prices (1997-2011)

This numerical example and figures are borrowed from [Otunuga and Ladde, 2014]. In this paper, the authors used the model for spot price with delayed stochastic volatility from the paper [Kazmerchuk, Swishchuk and Wu, 2005], and applied it to the Henry Hub daily natural gas data set for the period 02/01/2001- 09/30/2004. The data was collected from the United State Energy Information Administration website (www.eia.gov). From Figure 1 below we can see the properties of the gas daily spot prices: randomly driven, non-negative, mean reversion, jumps (spikes), unpredictability of spot price volatility:

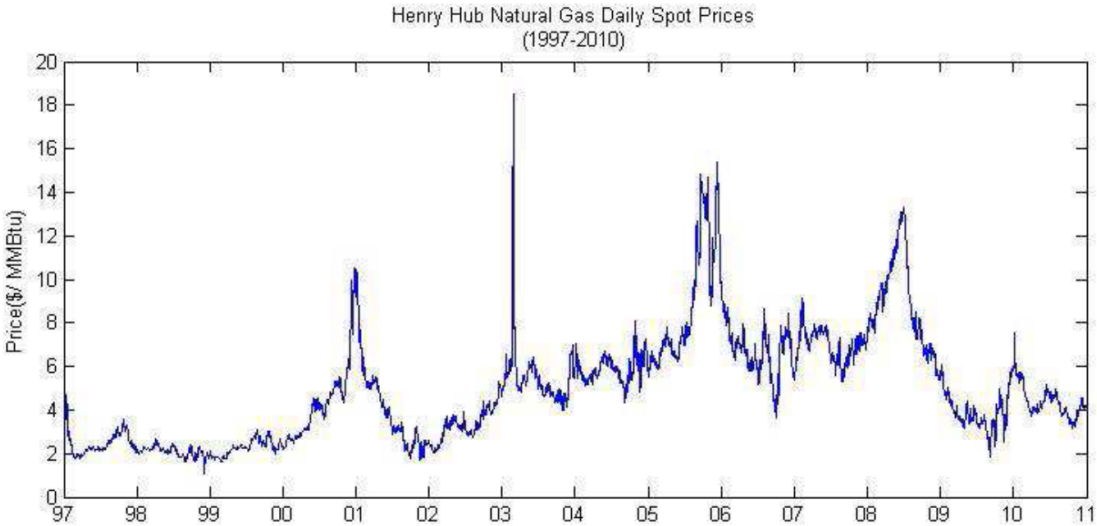


Figure 1: Plot of Henry Hub Daily Natural Gas Spot Prices (1997-2011)
(Courtesy-[Otunuga and Ladde, 2014])

Table 1 below gives descriptive statistics of Henry Hub Daily Natural Gas spot prices (1997-2011):

	Mean	Variance	Skewness	Kurtosis	Minimum	Maximum
S_t	4.9519	2.4966	1.0391	4.3491	1.05	18.48
$S_{t+1} - S_t$	-0.0001142	0.3189	-0.7735	191.8911	-8.01	6.50
$\ln(S_t)$	1.4754	0.5048	-0.0465	2.1540	0.0488	2.9167
$\ln[S_{t+1}/ S_t]$	2.8485e-5	0.0473	0.4814	22.0473	-0.56	0.5657

Table 1: Descriptive Statistics of Henry Hub Daily Natural Gas Spot Prices (1997-2011) (Courtesy-[Otunuga and Ladde, 2014])

As we can see from the Table 1 above, the logarithmic price is better than the raw price data because the variance for log is the smallest.

A simple model for the spot price is considered:

$$\ln S(t) = X(t),$$

where

$$dX(t) = \gamma(k - X(t))dt + \sigma(t, X(t))dB(t),$$

and

$$\frac{\sigma^2(t, X(t))}{dt} = [\alpha + \beta \int_{t-\tau}^{t} \sigma(s, X(s))dB(s)]^2 + c\sigma^2(t, X(t)).$$

The model for $\sigma^2(t, X(t))$ above is the same as the model for stochastic volatility with delay that we considered in [Kazmerchuk, Swishchuk and Wu, 2005].

Discrete scheme is implemented: $l = 2 = [\frac{\tau}{\Delta}]$, where Δ is the size of the mesh of the discrete-time grid, $[,]$ is the floor function.

Estimated Parameters are (Courtesy-[Otunuga and Ladde, 2014]):

γ	k	τ	α	β	c
1.8943	1.5627	0.008	0.433	-0.07	-1.5

Graphs below, Figure 2, includes Real, Simulated Spot Prices and Simulated Expected Spot Price (Henry Hub Daily Natural Gas Data Set (02/01/2001-09/30/2004)):

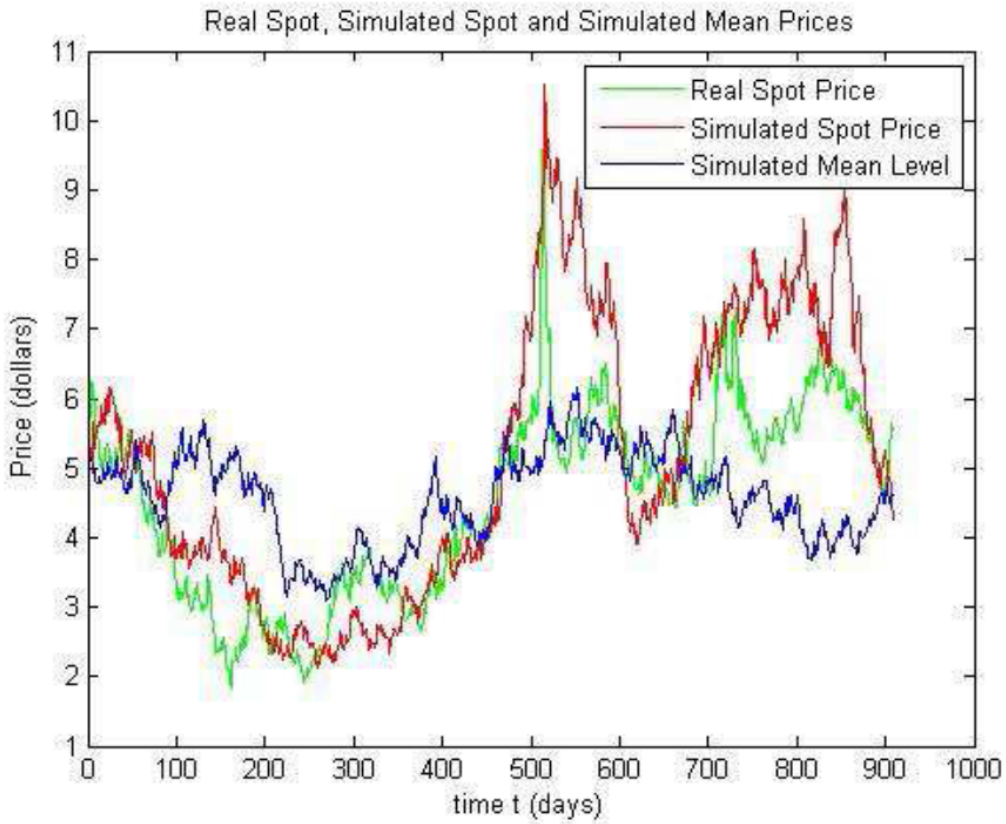


Figure 2: Real, Simulated Spot Prices and Simulated Expected Spot Price (Courtesy-[Otunuga and Ladde, 2014])

Graph below, Figure 3, shows simulated $\sigma(t, X(t))$ from Henry Hub Daily Natural Gas Data Set (02/01/2001-09/30/2004)).

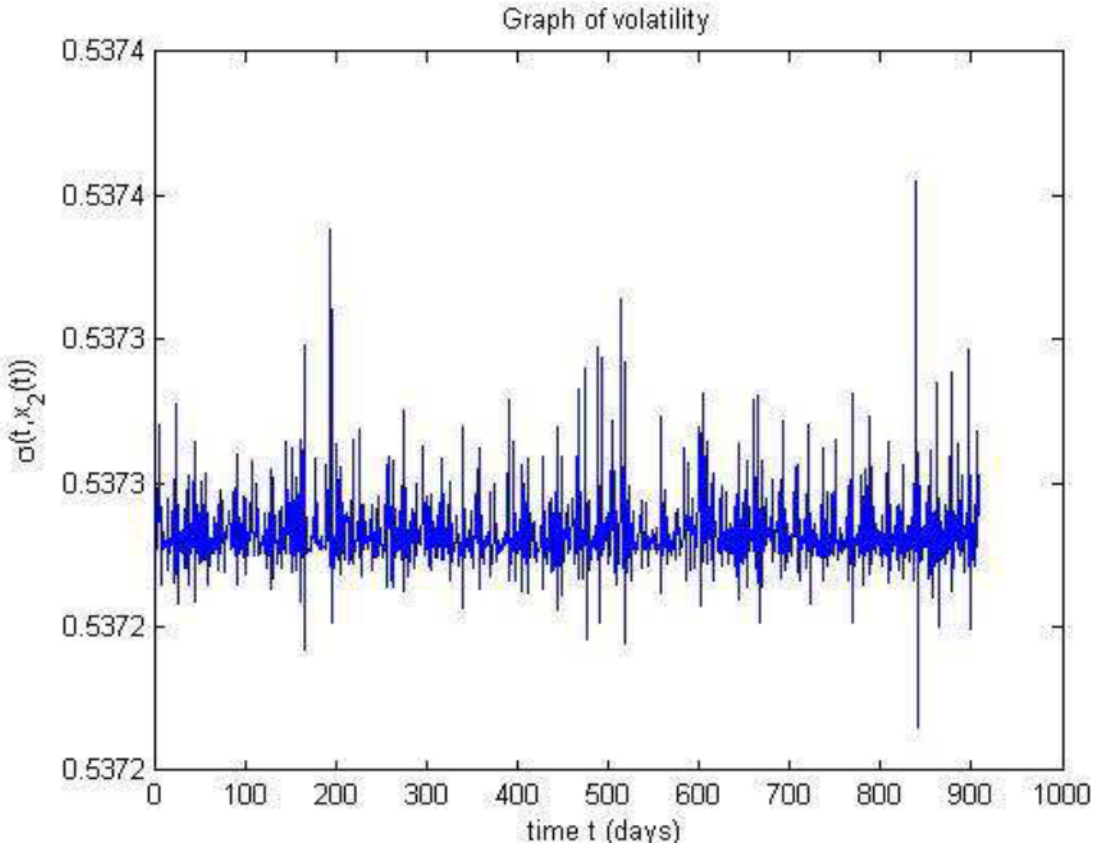


Figure 3: Simulated $\sigma(t, X(t))$ (Courtesy-[Otunuga and Ladde, 2014])

7 Mean-reverting Processes in Alberta Energy Markets Modelling [7]

The paper [7] is a bridge between fossil fuel energy research papers mentioned above, [1]-[6], and clean energy research papers which will be produced in near future.

In [7] paper we introduced a fuel-switching price to the Alberta market, which is designed for encouraging power plant companies to switch from coal to natural gas when they produce electricity; this has been successfully applied to the European market. Moreover, we consider an energy-switching price which considers power switch from natural gas to wind. We modelled these two prices using five mean reverting processes including a Regime-switching processes, Lévy-driven Ornstein–Uhlenbeck process and Inhomogeneous Geometric Brownian Motion, and estimate them based on multiple procedures such as Maximum likelihood estimation and Expectation-Maximization algorithm. Finally, this paper proves previous results applied to the Albertan Market where the jump modelling technique is needed when modelling fuel-switching data. In addition, it not only gives promising conclusions on the necessity of introducing Regime-switching models to the fuel-switching data, but also shows that the Regime-switching model is better

fitted to the data

Electricity prices are effected by factors such as weather conditions, economic growth, political principles and fuel-switching, where the fuel-switching is the only one that is controllable by the company itself. This paper [7] introduced a fuel-switching price, which explains how to switch from coal to natural gas when producing electricity, and how this can apply to the Albertan energy market. In the Albertan market, there are examples of power plant companies which are working on fuel switching, such as Transalta's Coal-to-Gas Conversions Project and Capital Power's Repowering Genesees Project. We also introduced an energy-switching price which considers a power switch from natural gas to wind. Moreover, note that this paper focuses on different jump and regime-switching mean-reverting stochastic models, where the financial and economic meanings and their applications will be discussed in another paper. We modelled these two prices using five mean-reverting processes and including regime-switching processes, such as the Lévy-driven Ornstein–Uhlenbeck process and inhomogeneous geometric motion. The estimation of these processes is based on multiple procedures such as maximum likelihood estimation and the expectation-maximization algorithm. This paper also justified previous results applied to the Albertan energy market showing that the jump modelling technique is needed when modelling fuel-switching data. Finally, the novelty of the paper lies not only in giving a promising conclusion on the necessity of introducing regime-switching models to the fuel-switching data, but also in showing that regime-switching models are better fitted to the data.

Thus, we considered five mean-reverting processes in this paper:

• Inhomogeneous geometric Brownian motion (IGBM):

$$dS_t = a(L - S_t)dt + \sigma S_t dW_t,$$

where $a, L, \sigma > 0$, $W_t$is a standard Brownian motion;

• OU process (OU):

$$dX_t = k(\theta - X_t)dt + \sigma dW_t,$$

where $k, \theta > 0$, W_t is a standard Brownian motion;

• Lévy-driven OU process (LDOU):

$$dX_t = k(\theta - X_t)dt + \sigma dL_t,$$

where $k, \theta > 0$, L_t is a Lévy process;

• Regime-switching OU process (RSOU)

$$dX_t = \kappa(Z_t)(\theta(Z_t) - X_t)dt + \sigma(Z_t)dW_t,$$

where Z_t is a continuous-time finite-state Markov chain, $k(z), \theta(z) > 0$ are bounded function of z.

- Regime-switching Lévy-driven OU process (RSLDOU):

$$dX_t = \kappa(Z_t)(\theta(Z_t) - X_t)dt + \sigma(Z_t)dL_t,$$

After the model comparison, we were able to conclude that the RSOU process and OU process are the best models for fuel-switching price and energy-switching price, respectively. We could see that for the fuel-switching price, the regime-switching model largely increases the goodness of fit compared to other models, which indicates the important property of regime-switching for this price. Moreover, jump modelling techniques are also important as they increase the performance of the OU process, and this finding is similar to the previous results from North American and European Markets. As reflected by the stochastic models, the fuel-switching price in the Albertan market includes jumps and regime-switching. However, as the natural gas price keep decreasing in Alberta, more and more companies have already switched their power plant to natural gas, and this is why we need to further consider energy-switching prices. The best fit of the OU process on energy-switching price reflects the steadiness of wind price, since it is a uniform distributed process.

8 Alternatives to Black-76 Model for Options Valuations of Futures Contracts [8]

In March 2020, the prompt month WTI futures contract settled below zero for the first time in the contract's history. Many market participants apply the Black-76 model or some variation when calculating the value of the options on this futures contract as a relatively straightforward, parametric valuation method. However, Black 76 requires positive underlying market prices. The negative prompt month settlement price caused considerable consternation among energy traders and risk managers.

More generally, OTC options are also available on basis or differential prices. These transactions are options on the difference between two published indexes such as NYMEX Henry Hub and AECO (for natural gas) or Cushing WTI and Houston (for crude oil). As such, these instruments frequently have negative underlying market prices.

Thus, our task was to propose alternative models to Black-76 to valuate option prices when the underlying future contracts can assume negative values.

In this paper [8], we proposed some alternatives to Black-76 model to value European options on future contracts in which the underlying market prices can be negative or/and mean reverting. We specifically consider two models, namely Ornstein-Uhlenbeck (OU), for negative prices, and continuous-time GARCH (or inhomogeneous geometric Brownian motion), for positive prices. We then analyze the results and compare them with

Black-76, the most commonly used model, when the underlying market prices are positive. Numerical examples are presented using WTI and NYMEX NG data sets.

Our methodology is the following one:

1. Take data (prices), sketch their behaviour, i.e., their evolution in time;

2. If the prices are positive and not mean-reverting, then use geometric Brownian motion (GBM) model for their evolution and Black-76 formula for option valuation of futures (see also formulas (BlCall) and (BlPut) in [12];

3. If the prices are positive and mean-reverting, then use continuous-time GARCH (or, another name, inhomogeneous GBM model) model [12] and option pricing formula (35) from [1], Theorem 5.1;

4. If the prices are both positive and negative, but not mean-reverting, then use Bachelier model and his formula (see formulas (Ba_1) and (Ba_2) below and in [12];

5. If the prices are both positive and negative, and mean-reverting with mean-reverting level 0, then use Ornstein-Uhlenbeck model and the formulas (OUCall_1) and (OUCall_2) below and from [12];

6. If the prices are both positive and negative, and mean-reverting with mean-reverting level non-zero, then use Vasicek model and the formulas (VasCall_1) and (VasCall_2) below and from [12].

In this paper we have shown how this methodology works on data sets presented by Scott Dalton (Ovintiv Services Inc.), namely, we used WTI and NYMEX NG data sets.

9 A Vision to Transition to 100% Wind, Water & Solar Energy in Canada [9]

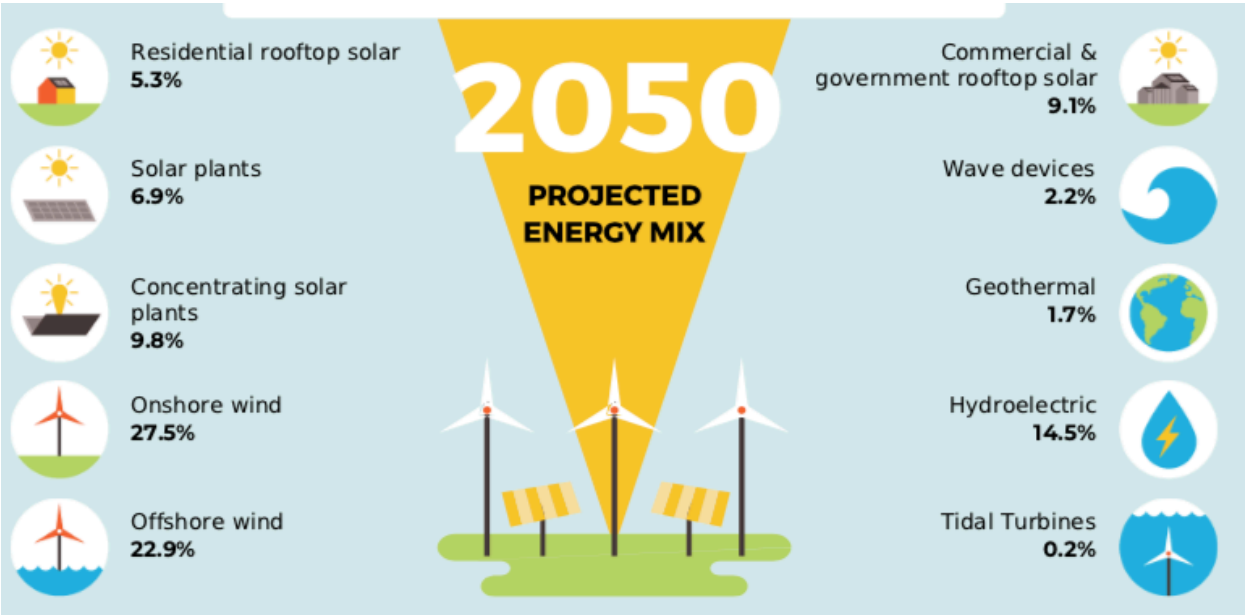
A group of U.S. civil engineering has calculated that Canada could be completely powered by renewable energy, if we just decide to do it.

They say that would save $110.1 billion on health care costs every year and prevent $9,884$ annual air pollution deaths.

Their research is available at thesolutionsproject.org.

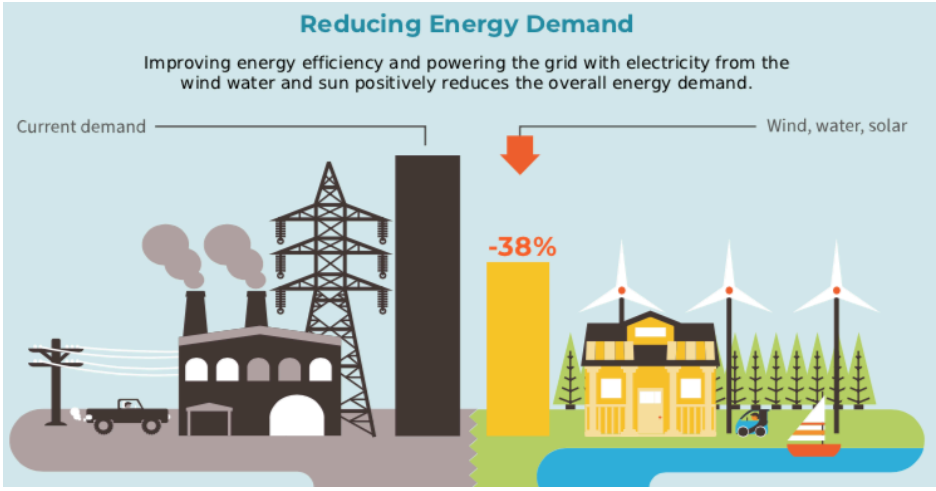
2050 **PROJECTED ENERGY MIX:**
-Onshore wind: 27.5%
-Offshore wind: 22.9%
-Hydroelectric: 14.5%
-Concentrated solar plants: 9.8%

-Commercial&government rooftop solar: 9.1%
-Solar plants: 6.9%
-Residential rooftop solar: 5.3%
-Wave devices: 2.2%
-Geothermal: 1.7%
-Tidal turbines: 0.2%



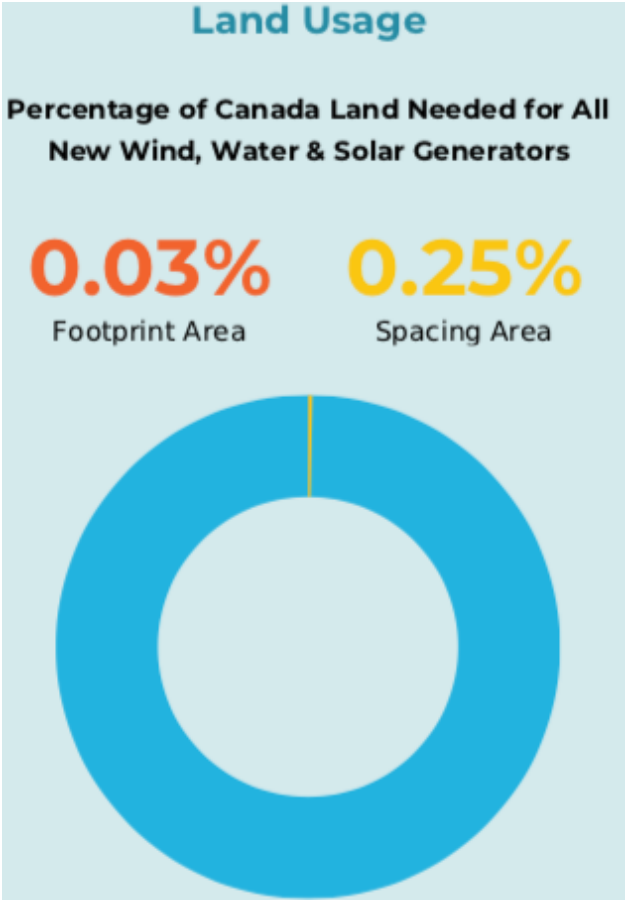
40 Year Jobs Created (Number of jobs where a person is employed for 40 consecutive years):
-Construction jobs: $315,138$
-Operation jobs: $367,889$

-Reducing Energy Demand: -38%
(by improving energy efficiency and powering the grid with electricity from the wind, water and sun positively reduces the overall energy demand)

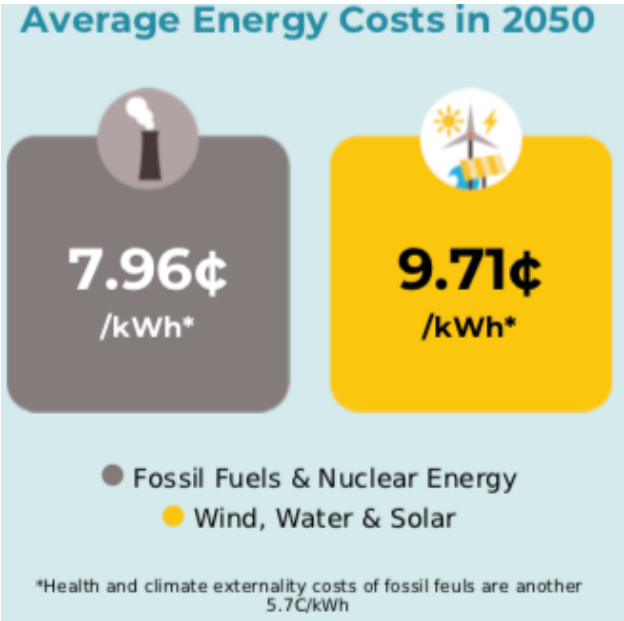


Health Cost Savings:
-Avoided health costs per year: $\$110.12B(3.94\%)$ of country GDP
-Lives lost to air pollusion that we could save each year: $9,884$

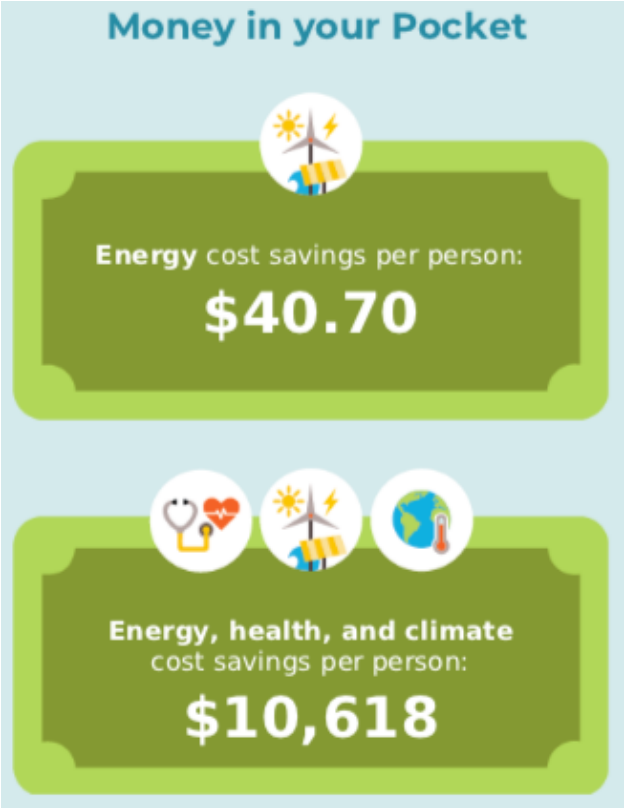
Land Usage (percentage of Canada Land Needed for All New Wind, Water & Solar Generators):
-Footprint Area: 0.03%; -Spacing Area: 0.25%



Average Energy Costs in 2050:
-Fossil Fuels & Nuclear Energy: 7.96¢
-Wind, Water & Solar: 9.71¢



Money in Your Pocket:
-Energy cost savings per person: $40.70
-Energy, health, and climate cost savings per person: $10,618



10 Wind and Solar Energy in Alberta [10]

Alberta could lead Canada in wind and solar power by 2025, expert says.
It forecasts that 83% of the combined utility-scale wind and solar capacity built in Canada over the next five years will be in Alberta. That wouldn't include smaller renewable development such as residential rooftop solar.
According to the data that Rystad tracks, Alberta's current renewable capacity includes 0.1 gigawatt (GW) of solar and 1.8 GW of wind. By 2025, it expects that to grow to 1.8 GW of solar and 6.5 GW of wind. Rystad forecasts Ontario will have about 1.8 GW solar and 5.8 GW wind in 2025.
Tan said Alberta's commitment to stop burning coal to generate electricity by 2030 "opens the door" for wind and solar to play a larger role.
We have 7693 Solar PV Systems in Alberta; 77% Residential, 11% Commercial, 11% Farm, 1% Other.
Alberta now ranks third in Canada for installed wind energy capacity.
Wind represents 9% of Alberta's total generation.
Alberta hydro electric facilities represent 5.5% of the market capacity of installed generation.
Travers Solar Project in Alberta is the size of 1,600 football fields and is making Alberta a leader in green energy. Amazon announced in June 2021 it will purchase power from a massive new solar farm in Alberta, marking the e-commerce giant's second renewable energy investment in Canada.
Construction began in the fall of 2020 on Travers Solar, a $\$700 - million$, 465-MW project southeast of Calgary, which its developers say will be the largest solar photovoltaic project in Canada and one of the largest in the world.
Privately held Greengate Power Corp. of Calgary has been working on the project for four years and is expected to have it completed by 2022. "It'll consist of 1.3 million solar panels spread over more than 3,000 acres (1,215 hectares) of farmland," said Dan Balaban, CEO of Greengate Power. "And it'll produce a sustainable source of energy for more than 150,000 homes."

11 Energy Transition Center in Calgary, AB, Canada [11]

$\$2.14M$ investment announced to support energy transition collaboration between UCalgary ecosystem and energy industry. The Energy Transition Centre (ETC) is a three-year project with a budget of 17.5 million. Energy Transition Centre in downtown Calgary will support clean energy startup companies.

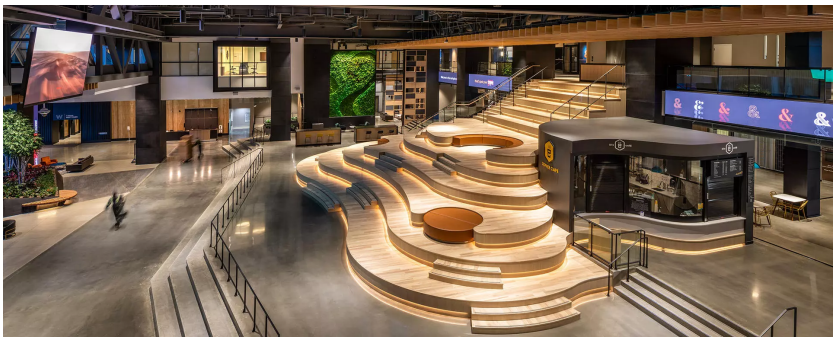
The University of Calgary ecosystem is poised to lead the energy transition with an investment announced this week from the Government of Canada. Prairies Economic Development Canada (PrairiesCan) is investing $2,140,205$ to support the Energy Transition Centre (ETC) in downtown

Calgary.

The Energy Transition Centre is expected to support innovative clean energy development and generate economic activity through new business opportunities and research and development, while also assisting the commercialization of technologies for industry. Over the next three years, the initiative expects to create 25 new small- and medium-sized firms while assisting an additional 25 existing firms in accelerating their technologies for the clean technology sector. The ETC is a collaboration between the University of Calgary, Innovate Calgary, Avatar Innovations, and the energy industry. It is expected that programming at the ETC will engage highly qualified personnel from both the academic and industry sectors through career development, as well as technology development in areas of emerging technologies crucial for the energy transition. This includes engaging and supporting diverse industry employees as well as those involved in university research.

The ETC is designed to encourage energy transition solutions by providing programming that focuses on a mass upskilling of energy workers. This will be achieved through a training curriculum that cultivates cross-learning between energy professionals and university postdocs. Programs will also nurture transformative technologies through curriculum that entwines both business and technology de-risking components and provides access to technical experts and capital markets for commercialization.

Ampersand building in downtown Calgary, site of the Energy Transition Centre



Gathering space at the Energy Transition Centre.

12 Conclusion and Future Work

The paper overviewed our recent results in energy market modelling, including: option pricing formula for a mean-reversion asset; variance and volatility swaps in energy markets; applications of weather derivatives in energy markets; pricing crude oil options using Levy processes; energy contracts modelling with delayed and jumped volatilities; applications of mean-reverting processes in Alberta energy markets. We will also considered the clean renewable energy prospective in Canada, and, in particular in Alberta and Calgary. The future research work will be devoted to the renewable energy markets modelling, including wind, solar and water energy modelling.

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